PREFACE

Two men look out through the same bars:

Galactic Ionised Hydrogen Condensations

Frederick Langbridge (1896)

In this volume both the mud and the stars get their share of attention; the stars are the sources of the energy received by radio-telescopes from the clouds of ionised gas which first part of the dissertation, and the mud in Cambridgeshire blue clay, which insulates many miles of radio cables from the changes of temperature with which the final two chapters are preoccupied.

The tenuous link between these two rather disparate topics is the Cambridge Radio Telescope with which I have been fortunate enough to be associated during the past two years as a research student in the Cavendish Laboratory. The greater part of my research during that time has consisted of analysing observations made with the instrument, in my case, of Galactic HII regions and planetary

A Dissertation

Trinity College

Cambridge

August 1969
"Two men look out through the same bars:
One sees the mud, and one the stars."

Frederick Langbridge (1896)*

In this volume both the mud and the stars get their share of attention; the stars are the sources of the energy received by radiotelescopes from the clouds of ionised gas which are studied in the first part of the dissertation, and the mud is Cambridgeshire blue clay, which insulates many miles of radio cables from the changes of temperature with which the final two chapters are preoccupied.

The tenuous link between these two rather disparate topics is the Cambridge One Mile Radio Telescope with which I have been fortunate enough to be associated during the past two years as a research student in the Cavendish Laboratory. The greater part of my research during that time has consisted of analysing observations made with the instrument, in my case, of Galactic HII regions and planetary nebulae. This work, described in Chapters I - VI, comprises

* See page 152
the main body of this dissertation and provides its title. The final two chapters are concerned with some of the difficulties in applying the principles used in the One Mile Telescope to the longer and more elaborate Five Kilometre Telescope which is now under construction at Cambridge. In particular I have been studying theoretically and experimentally the measurement of and variations of the lengths of the radio frequency cables connecting the component aerials. In addition there are two appendices; the first describes some computer programs which I have written and used at various points in the work, and the second contains summaries of the relevant preprints and private communications I have received during the final month before submitting this dissertation, mainly on the source NGC 6857.

A great deal of the contents of this dissertation is new and most has not yet appeared in print. I have made every effort to indicate the source of any data which is not original in the text. The analysis of the four sources in Chapters II - V is solely my own work, as are my thoughts on their evolution in Chapter VI, though naturally I have benefitted greatly from the advice of many other members of the department on both general and specific points. This is especially true of Chapters II
and IV since I have already had papers published on W 49 and NGC 6857. During the course of their being written innumerable helpful, and a few unhelpful, suggestions were made in the draft stages for improving both the science and the style and many such improvements have been incorporated into the chapters on these sources. Although no paper has yet been submitted for publication much of Chapter VII has been drafted into a probable future paper and a number of Professor Ryle's suggestions about this work have been gratefully accepted. None of the other chapters have been circulated at any stage, however.

Working with an instrument as complex and as expensive as the One Mile Telescope must inevitably involve a great deal of cooperation and I am very grateful to very many other members of the department for assistance during making observations and advice while reducing them or building equipment. In particular I must thank Professor Ryle and my supervisor Dr. Kenderdine for their continued support and encouragement. I would also like to thank Miss Dunn and Mrs. Ashton for so painstakingly preparing most of the diagrams, and Mrs Petrie for assistance with computing and for Always Knowing Where Things Are. Finally I am grateful to the Science Research Council for providing the necessities of life for the past two years.
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I. THE STUDY OF CONDENSATIONS IN HII REGIONS

1. HII Condensations as a Class of Radio Source.

In this dissertation observations of four radio sources are described, W 49, DR 21, NGC 6857 and NGC 7027. All the sources are inside our Galaxy and emit radio noise by thermal emission from ionised hydrogen, though W 49 has, in addition, a non-thermal component. The sources differ very much but all four are, or contain, compact condensations of dense ionised gas (Diameter \(< 0.3 \) pc, \( N_e \geq 10^4 \) cm\(^{-3}\)). NGC 7027 is a planetary nebula but the condensations in the other sources are almost certainly excited by young OB type stars.

Although DR 21 was the first such object to be studied in detail (Ryle and Downes 1967) the recognition of these condensations as an important new class of radio source was contained in a paper by Mezger et al. (1967b). These authors found evidence for compact HII regions, chiefly recognisable by their high turnover frequencies, in the diffuse HII regions W 49, W 75 (DR 21) and W 3. In all cases intense, non-thermal OH emission was found near to, or coincident with the condensations, and, in the case of DR 21, the only source whose H109α recombination line was
readily observable, the unusually high value of 25 km/sec was found for the R.M.S. turbulent velocity, suggestive of rapid expansion processes. Mezger et al. (1967b) put forward the hypothesis that these condensations are formed when a cloud of neutral gas and dust contracts to form a star whose ultraviolet radiation then excites part of the remainder of the cloud giving rise to what Davidson and Harwit (1967) have termed a "cocoon star". Such an object would be obscured at optical wavelengths owing to the density of the dust surrounding the star. Theory also predicted that the dust would cause the objects to be strong emitters in the infra-red, although this is not yet reliably confirmed by observations.

Microwave formaldehyde absorption (Snyder et al. 1969) and intense $\text{H}_2\text{O}$ emission (Cheung et al. 1969) have been discovered in W 49, W 75 and some other HII regions indicating the presence of unusual physical conditions in these localities. Indeed Hughes (1969) has suggested that the condensations themselves are protostars and that the emission is by synchrotron radiation. Reasons for not accepting this hypothesis are given in Chapter III, however.

The Cambridge One Mile Telescope, the source of all the new data in this dissertation, is almost the only instrument able to resolve the structure of these
fascinating objects. High resolution is vital as the condensations sometimes exist in groups and cannot be studied individually with the resolving power available with a single dish. It is shown in Chapter VI how determination of their sizes has led to a preliminary understanding of the evolution of these condensations, and to tentative, but important, conclusions about stellar formation. The data on which these hypotheses are based would have been very difficult to obtain by any other means. The information which has been gained from the present studies of the four sources is quite varied as each source has presented different problems and displayed different points of interest. A summary of the observations is presented in the next section, and detailed accounts of each source in Chapters II – V.

2. The Sources Observed.

The four sources, which are described briefly in this section, were studied at various frequencies between 408 and 4995 MHz. The One Mile Telescope (Ryle 1962) was designed to operate at two of these simultaneously but changing from one pair of frequencies to another is a lengthy procedure. For this reason all observations made with the telescope before the summer of 1968 were at 408
and 1407 MHz ($\lambda = 75$ and 21 cm) and all those since have been made at 2695 or 4995 MHz ($\lambda = 11$ cm or 6 cm). Unfortunately, for technical reasons, simultaneous operation at the latter two frequencies has not been possible up to now.

Table 1 shows the frequencies at which the observations described in this dissertation were made, and gives the half-power width of the telescope beam in right ascension for each frequency. Before the author joined the group the One Mile Telescope had been used at 408 MHz and 1407 MHz to observe DR 21 (Ryle and Downes 1967) and NGC 7027 (Elsmore 1968); the new data are summarised below.

W 49 (Chapter II) was the first source to be observed, and consists of an HII region and a supernova remnant close together. The HII region is one of the largest in the Galaxy and was shown to be double with an unusually high electron temperature, while the supernova remnant was shown to have a shell structure with several unusual features. A study of the high density condensations themselves was not possible in the case of W 49 because of self absorption at the frequencies used, but pilot observations at 5 GHz confirmed their presence. Most of the contents of Chapter II have already been published (Wynn-Williams 1969a).

DR 21 (Chapter III) might be described as the prototype
<table>
<thead>
<tr>
<th>Source</th>
<th>Frequency (MHz)</th>
<th>408</th>
<th>1407</th>
<th>2695</th>
<th>4995</th>
</tr>
</thead>
<tbody>
<tr>
<td>W 49</td>
<td>X</td>
<td>X</td>
<td>-</td>
<td>X (partial)</td>
<td></td>
</tr>
<tr>
<td>DR 21</td>
<td>RD</td>
<td>RD</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>NGC 6857</td>
<td>-</td>
<td>-</td>
<td>X</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>NGC 7027</td>
<td>E</td>
<td>E</td>
<td>-</td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>Half Power Beamwidth (sec arc)</td>
<td>80''</td>
<td>23''</td>
<td>12''</td>
<td>6.5''</td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Frequencies of observations and telescope beamwidths in right ascension. X denotes observations by the author. Previous studies of the sources with the One Mile Telescope are by Ryle and Downes (1967) and Elsmore (1968), denoted by RD and E respectively.
compact HII region, as it is a bright, single source with
the largest angular size of any of the condensations known.
In the new observations it was possible to make a direct
estimate of its electron temperature as well as study the
distribution of gas in the source.

NGC 6857 which, as will be shown in Chapter IV, is
associated with the diffuse HII region W 58, is probably
the most interesting of the four sources studied as far as
new data are concerned. NGC 6857 was thought to be a
planetary nebula until the present observations were made,
and the proposal that it is in fact one of a group of
compact, young HII regions was first made in a paper by the
author (Wynn-Williams 1969b). One of the four compact
sources has properties very similar to those of DR 21 and
the other three are thought to represent similar objects
at a later stage of evolution. If this proposal is correct
then optical studies of HII condensations are possible for
the first time since two of the four components of W 58,
including the most compact one, are associated with known
emission nebulae. Also the spatial relationship of a
group of condensations is displayed for the first time.
The existence of four components, all with measurable para-
meters, has greatly increased the total available data on
HII condensations and the discussion of their evolution in
Chapter VI relies greatly on this information. It is unfortunate that it was possible to make observations of NGC 6857 at only one frequency, but the overall size of the source means that 16 days observing time would be required to map the source at 5 GHz. Because of shortage of observing time on the telescope this has not so far been feasible, and lower frequency observations will have to wait until the system is next converted from 2.7 and 5 GHz, which will be many months.

NGC 7027 (Chapter V), a very bright planetary nebula, was observed partly for comparison with HII condensations and partly because it is a very interesting source in its own right, having a remarkable infra red excess. At 5 GHz it is sufficiently resolved for useful comparisons to be made with the optical data. It was not observed at 2.7 GHz as it would have been unresolved at that frequency.

Finally, in Chapter VI all the data on compact HII regions are summarised, their evolution and significance are discussed, and proposals are made for future observations.


The operation of the One Mile Telescope, which consists of one movable and two fixed 18m paraboloids, has been fully described elsewhere (Elsmore et al. 1966; Macdonald 1968).
The aperture which is synthesised by the instrument consists of a series of concentric circular annuli lying parallel to the equatorial plane. At 1407 MHz the response pattern is a pencil beam with half-power widths 23" arc in right ascension and 23"cosec δ in declination. The use of a partly filled aperture results in the additional presence of elliptic grating rings concentric with the main beam. If n positions of the moving aerial are used for the synthesis, giving rise to 2n spacings altogether, the radius of the primary grating ring is approximately 60n" arc in right ascension and 60n"cosec δ in declination at 1407 MHz. Proportionate values are obtained at other frequencies. The number of positions used in a survey is generally the least required to ensure that the radius of the first grating ring is larger than the overall diameter of the source. Under some circumstances grating rings can be removed from synthesised maps, and, as described in Chapters II and IV this was necessary to a greater or lesser extent for W 49 and NGC 6857. Otherwise the data reduction closely follows that described by Elsmore et al. (1966), the necessary Fourier Transforms being performed on the TITAN computer of the University Mathematical Laboratory.

The output consists of maps with contours of equal surface brightness. Two methods of presentation of these
maps have been used in this dissertation, the difference being connected with the fact that the beam of the telescope is elliptical. For W 49, DR 21 and NGC 7027 "aerial coordinates" have been used: in the maps of these sources the declination scale has been multiplied by a factor $\sin \delta$. This makes it easier to determine whether or not fine structure has been resolved by the telescope since the beam then has a circular cross section. This is the usual way One Mile Telescope results are presented. In the case of NGC 6857, however, "sky coordinates" have been used, in which the scales are the same in both right ascension and declination and the beamshape is elliptical. This is convenient when comparison with optical data is important. Throughout the dissertation coordinates are (1950.0).
II. THE GALACTIC RADIO SOURCE W 49

1. Introduction.

W 49 (3C398) is a radio source on the Galactic Equator consisting of two components separated by about 12' arc. The region is heavily obscured, yet despite the absence of optical data, W 49 is of unusual interest because radio observations have shown that it consists of an HII region (A), associated with which are two centres of intense OH emission (Rogers et al. 1967) and a non-thermal component (B) which may be an old supernova remnant (Mezger and Henderson 1967). Since the present observations were completed W 49 has been discovered to be the most intense source of cosmic H₂O emission known (Cheung et al. 1969) and also to exhibit strong formaldehyde absorption (Snyder et al. 1969). From studies of the Doppler shift of its 5 GHz H109α recombination line Mezger and Höglund (1967) deduced that the thermal component is at a distance of 14.1 kpc. This estimate is in agreement with the HI absorption studies of Bystrova et al. (1968), of Sato et al. (1967) and more recently Sato (1969). In the latter two papers, however, it is suggested that component B is about 1 kpc nearer the sun than component A, making them
physically unrelated, but this has been disputed by Mezger et al. (1967a).

Most of the contents of this chapter has already appeared in print (Wynn-Williams 1969a). Previously maps of the source had been published by Mezger and Henderson (1967) at 5 GHz, Mezger et al. (1967a) at 0.611, 1.414 and 15.4 GHz, and by Hughes and Butler (1967) at 10.5 GHz. In the maps at the two highest frequencies, which had resolutions of 2' and 2.8' arc respectively, the thermal component showed signs of having a complex structure, being extended along a line approximately joining the two centres of OH emission. From this evidence and from a study of the spectrum of the source Mezger et al. (1967a) concluded that the HII region consists of a fairly low density envelope containing several high density condensations which become optically thick at about 7 GHz and hence contribute little to the total flux below 2 GHz.

Since W 49 is one of the more distant galactic sources it has a comparatively small angular size which has made it a difficult object for detailed study at intermediate frequencies, where both components have large flux densities and the main part of the HII region is becoming optically thick. In this chapter results of observations at 408 and
1407 MHz made with the One Mile Telescope are presented, which have allowed examination of the structure of the source with far greater resolution than has hitherto been possible. In addition the results of a preliminary survey at 5 GHz are mentioned.

2. Observations.

The operation of the telescope has been outlined in Chapter I. The observations of W 49 were made in the autumn of 1967 at 408 and 1407 MHz simultaneously. Because of the low declination of W 49 ($\delta = 9^\circ$) the beam is greatly elongated in the north-south direction, having an elliptical section 23" x 148" at 1407 MHz and 80" x 510" at 408 MHz. Since the overall extent of W 49 is about 15' arc 32 spacings of the aerials were required to ensure that at 1407 MHz the innermost grating rings from one component did not confuse the other. Figure 1 shows the results obtained from the data in the form of maps with contours of equal surface brightness. "Aerial coordinates" have been used in this case (see section I 3) with the declination scale multiplied by $\sin \delta$. This makes it easier to determine whether or not fine structure has been resolved by the instrument, since the beam then has a circular cross section. However at this low declination $\sin \delta = 0.156$ and the process results
FIG. 1. W 49 at 1407 MHz and 408 MHz. The thermal component (A) is on the right with the positions of regions of OH emission marked on the 1407 MHz map. The shaded circles represent the half power beamwidths. The vertical scale is compressed by a factor 0.156 as explained in section 2. The contour intervals are 49°K at 1407 MHz and 126°K at 408 MHz.
in considerable distortion of fully resolved features.

Since the angular extent of W 49 is an appreciable fraction of the response of a single 18m paraboloid (54' arc at 1407 MHz) a correction for the appropriate reduction in gain was made to each point of the map before the contours were drawn. In addition, the use of 32 spacings for the synthesis resulted in the first grating ring having a radius of 16' arc in right ascension at 1407 MHz and 54' arc at 408 MHz. In the former case traces of these rings were visible on the original map near the outer edges of the two components. However since most of the structure lies along an east-west line, the rings were of constant cross section over the declination range of interest. Thus the profile of the rings could be calculated at declinations north and south of the source and a small correction applied to the maps. The modified contours were then redrawn manually.

The flux densities of the various components were obtained by numerical integration of the brightness distribution, corrected for the response of the paraboloids. A further correction was necessary because of the automatic gain control used in the phase switched receivers, which is arranged so that the sum of the aerial noise and receiver noise is kept constant at the output of the IF amplifiers.
The receiver noise temperature at 408 MHz is about 130°K and at 1407 MHz is about 200°K, so that during observations at low galactic latitudes the output sensitivity is reduced compared to that when observing the calibration source 3C 295, since W 49 is in a region of the Galaxy which has a brightness temperature of 130°K at 408 MHz (Pauliny-Toth and Shakeshaft 1962). To evaluate this correction, a separate measurement was made in which the automatic gain control of one receiver was switched off and the relative total noise temperature for the area of sky around W 49 and for a nearby region of sky outside the galactic plane was recorded by pointing the dish at the two regions alternately and monitoring the detector current in the receiver. This experiment showed that the measured brightness had to be increased by factors of $1.43 \pm 0.04$ at 408 MHz and of $1.14 \pm 0.03$ at 1407 MHz.

The final values for the flux densities of the main components are shown in Table 2. These values are considered accurate to about $\pm 15\%$ and have been used to calculate the mean spectral index over the range, using the sign convention:

$$S_\nu \propto \nu^{-\alpha}$$

The values for the thermal component are given separately for the two principal components and for the
<table>
<thead>
<tr>
<th>Source</th>
<th>Flux density (10^{-26} W m^{-2} Hz^{-1})</th>
<th>Spectral Index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S_{1407}$</td>
<td>$S_{408}$</td>
</tr>
<tr>
<td>Thermal Source A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_{51}$</td>
<td>25</td>
<td>8</td>
</tr>
<tr>
<td>$A_{59}$</td>
<td>8</td>
<td>36 $\pm$ 5</td>
</tr>
<tr>
<td>Easterly Extension</td>
<td>3</td>
<td>36 $\pm$ 5</td>
</tr>
<tr>
<td>Non-thermal Source B</td>
<td>31 $\pm$ 5</td>
<td>59 $\pm$ 9</td>
</tr>
<tr>
<td>Total</td>
<td>67 $\pm$ 10</td>
<td>85 $\pm$ 13</td>
</tr>
</tbody>
</table>

Table 2. Flux densities of W 49
long easterly extension towards R.A. 19° 08' 16". The
two bright components are referred to as $A_{51}$ and $A_{59}$
according to their R.A. position in seconds. This is
to avoid confusion with the differing uses of the designa-
tions $A_1$ and $A_2$ by Mezger et al. (1967a) and Hughes
and Butler (1967).

Agreement between these results and those of
previous workers is quite good, except that Pauliny-Toth
et al. (1966) and Mezger et al. (1967a) obtained values of
$47.2 \pm 1.8$ and $44 \pm 4 \times 10^{-26} \text{ W m}^{-2} \text{Hz}^{-1}$ for the flux density
of component $A$ near 1400 MHz as opposed to the value of $36 \pm 5$
found in the present observations. Since, however, the
integrated flux densities from both components together
agree within experimental errors, the discrepancy may be
due to inaccurate estimation of the relative contributions
of the two components at the lower resolving powers previously
attainable. This is not unlikely in view of the fact that
both components are now seen to have a very complicated
structure and that there are steep gradients in the sky
brightness temperature in this region. In these new
observations, however, the resolution is sufficient to distin-
guish between the components completely, whilst errors caused
by variations in background temperature are very small.

The components are discussed in the following sections.
3. The Thermal Component W 49A

The radiation from the thermal component (A) has two main peaks of emission whose positions are very close to those of the two intense regions of OH emission (Rogers et al. 1967) as may be seen from Table 3.

Both these HII regions are partially resolved at 1407 MHz in right ascension, but only the areas to the west of \(A_{51}\) and the east of \(A_{59}\) are significantly broadened in declination. The large source, \(A_{51}\), clearly has a more complex structure, which might possibly consist of three components of unequal intensity, or a bright region surrounded by an uneven halo. There is, in addition, a considerable extension towards W 49B, about 4' arc in length. Since the surface brightness of this region is much lower the width of the extension is difficult to determine, owing to fluctuations in the background level of the map, but is probably of the same order as the rest of the source. Table 4 shows the measured widths of the components and their Gaussian widths (\(\theta_g\)) calculated from the formula:

\[
\theta_g^2 = \theta_{obs}^2 - \theta_a^2
\]  

(1)

where \(\theta_{obs}\) is the observed width and \(\theta_a\) is the width of the aerial beam. In declination a source width of 70'' represents a broadening of 10%, which is about the
<table>
<thead>
<tr>
<th></th>
<th>HII position</th>
<th>OH position</th>
</tr>
</thead>
</table>
| A51 | 19^h 07^m 51.3 ± 0.2^s  
09° 01' 20'' ± 20'' | 19^h 07^m 49.7 ± 1^s  
09° 01' 12'' ± 5'' |
| A59 | 19^h 07^m 58.8 ± 0.2^s  
09° 00' 10'' ± 20'' | 19^h 07^m 58.2 ± 1^s  
09° 00' 04'' ± 5'' |

**Table 3.** Positions of peaks of HII continuum emission (1950.0) compared with positions of OH emission given by Rogers et al.
In order to determine the optical depths, and hence the physical conditions, in the different components, the emission function was expanded to the sum of Gaussians. These distributions were convolved with the instrumental function to give the apparent optical depth at the appropriate frequency. The resulting integral was fitted with a distribution of a Gaussian, which was then integrated, divided by the magnitude of the instrumental function, and the apparent optical depth was computed. The resulting optical depths are given below.

<table>
<thead>
<tr>
<th></th>
<th>$A_{51}$</th>
<th>$A_{59}$</th>
<th>Easterly Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured widths $\Delta \alpha$</td>
<td>130&quot;</td>
<td>45&quot;</td>
<td>$\sim 240&quot;$</td>
</tr>
<tr>
<td></td>
<td>160&quot;</td>
<td>150&quot;</td>
<td>165&quot;</td>
</tr>
<tr>
<td>Measured widths $\Delta \delta$</td>
<td>$\lessapprox 70&quot;$</td>
<td>$\lessapprox 70&quot;$</td>
<td>$\sim 70&quot;$</td>
</tr>
<tr>
<td>Gaussian widths $\Delta \alpha$</td>
<td>130&quot;</td>
<td>40&quot;</td>
<td>$\sim 240&quot;$</td>
</tr>
<tr>
<td></td>
<td>$\lessapprox 70&quot;$</td>
<td>$\lessapprox 70&quot;$</td>
<td>$\sim 70&quot;$</td>
</tr>
<tr>
<td>Emission Measure (pc cm$^{-6}$)</td>
<td>$1.7 \times 10^6$</td>
<td>$0.8 \times 10^6$</td>
<td>$7 \times 10^4$</td>
</tr>
<tr>
<td>Assumed diameter (pc)</td>
<td>8.4</td>
<td>4.2</td>
<td>$4.2 \times 16.5$</td>
</tr>
<tr>
<td>Electron Density (cm$^{-3}$)</td>
<td>450</td>
<td>450</td>
<td>$\sim 130$</td>
</tr>
<tr>
<td>Mass (M$\odot$)</td>
<td>4500</td>
<td>560</td>
<td>$\sim 680$</td>
</tr>
<tr>
<td>Excitation Parameter (pc cm$^{-2}$)</td>
<td>250</td>
<td>125</td>
<td>$\sim 85$</td>
</tr>
</tbody>
</table>

Table 4. Measured sizes and calculated physical parameters of the different components of W 49A

Figure 21. Integrated profiles of W 49A at 408 MHz (continuous line) and at 1407 MHz after numerical smoothing (dashed line). The positions of the centres of components $A_{51}$ and $A_{59}$ are shown.
variation expected from noise and other errors.

In order to determine the optical depths, and hence the physical conditions, in the different parts of the source, the 1407 MHz map was convolved with a smoothing function so as to give it the same resolution as at 408 MHz. The brightness distribution, integrated over the declination range of the source, is plotted as a function of right ascension in Figure 2. The areas under the curves are proportional to the integrated flux densities at the appropriate frequencies and the positions of the peaks $A_{51}$ and $A_{59}$ are marked. In view of the fact that the separation of these peaks is little more than one beamwidth, and that the components themselves are dissimilar in both shape and brightness, it is impractical to make a quantitative estimate of their optical depths separately. Figure 2 does suggest, however, that $A_{59}$ has a rather smaller optical depth than $A_{51}$; if both components have similar temperatures and densities this result is consistent with its smaller size. The easterly extension is clearly of small optical depth.

FIG. 2. Integrated profiles of W 49A at 408 MHz (continuous line) and at 1407 MHz after numerical smoothing (dashed line). The positions of the centres of components $A_{51}$ and $A_{59}$ are shown.
A comparison of the curves in Figure 2 suggests that $A_{51}$ contributes about $16 \times 10^{-26} \text{ cm}^{-2} \text{ s}^{-1}$ at 408 MHz. Its temperature may be estimated by using the relation:

$$T \approx \frac{2.8 \times 10^7 \nu^2}{\Omega}$$  \hspace{1cm} (2)

at a frequency at which the source is optically thick. To apply the formula accurately requires measuring $\Omega$, the solid angle subtended by the source, at this higher frequency (with good resolution) and then using the flux density at a lower frequency to determine the temperature. Accurate estimation of $\Omega$ is difficult. In an earlier analysis (Kynman et al. 1969a), $A_{51}$ was assumed to be elliptical with diameter $130'' \times (70'' \times 1.471$). The factor $1.471$ is from Maunder and Henderson (1957) and An allowance for the fact that the radio width of a spherical distribution of gas of unit density is narrower than that of the comparable Gaussian distribution was made. This method gave $T \approx 3$ square and $T \approx 12,000 \text{ K}$. Subsequently, during the analysis of NO 7027 and M 21 (Chapters III and X), more careful modelling procedures were experimented with (Appendix A), and it was found that using Maunder and Henderson's factor would lead to an underestimation of the size of the source. This is because the application of equation (1) is inaccurate if
A comparison of the curves in Figure 2 suggests that $A_{51}$ contributes about $16 \times 10^{-26}$ W m$^{-2}$Hz$^{-1}$ at 408 MHz. Its temperature may be estimated by using the relation:

$$S_v \lesssim \frac{2kT_e v^2 \Omega}{c^2}$$

(2)

at a frequency at which the source is optically thick. To apply the formula essentially requires measuring $\Omega$, the solid angle subtended by the source, at the higher frequency (with good resolution) and then using the flux density at a lower frequency to determine the temperature. Accurate estimation of $\Omega$ is difficult. In an earlier analysis (Wynn-Williams 1969a) $A_{51}$ was assumed to be elliptical with diameters 130" x (70" x 1.471). The factor 1.471 is from Mezger and Henderson (1967) and is an allowance for the fact that the radio width of a spherical distribution of gas of uniform density is narrower than that of the comparable Gaussian distribution. This method gave $\Omega = 3$ sq.'arc and $T_e \gtrsim 12,000$ °K.

Subsequently, during the analysis of NGC 7027 and DR 21 (Chapters III and V), more careful model fitting procedures were experimented with (Appendix A), and it was found that using Mezger and Henderson's factor could lead to an underestimation of the size of the source. This is because the application of equation (1) is inaccurate if
the source profile itself is not Gaussian. It was found that for a sphere of uniform density with intermediate optical depth 10% broadening was produced by a source with diameter 75% of the half power beam. This means that the width in declination of $A_{51}$ may be as much as 110" arc, which is quite close to the diameter in right ascension. The electron temperature using this parameter turns out to be 11000°K.

The ratio of the flux densities, however, indicates that the source is not completely optically thick at 408 MHz. Mezger et al. (1967a) have tabulated a function derived by Osterbrock (1965) giving the spectrum of a spherical HII region near its turnover frequency. Although not entirely applicable because of the uncertainty in the shape of $A_{51}$, the results suggest an optical depth of 2.0 at 408 MHz and a turnover frequency, at which $\tau = 1$, of 570 MHz. This corresponds to an electron temperature of 16000°K. It has been pointed out, however, that the present value for the flux density of W 49A at 1407 MHz is about 25% lower than that determined by other observers. If the flux density is significantly greater than $36 \times 10^{-26}$ W m$^{-2}$Hz$^{-1}$ the correction factor for finite optical depth at 408 MHz will be overestimated and the electron temperature will not be as high as 16000°K. Nevertheless the
temperature is higher than is usual for diffuse HII regions and is in the range normally associated with planetary nebulae. A lower value for $T_e$ can only be accepted if part of the radiation at 408 MHz is being emitted from an area larger than 3.3 sq. arc. This would be possible if the bright peaks $A_{51}$ and $A_{59}$ were surrounded by an envelope of low brightness with spectral index close to zero which would be contributing proportionally more of the radiation at 408 MHz; such a component might represent a continuation of the optically thin easterly extension. A comparison of the outlines of components A and B at 1407 MHz (Figure 1) shows that the thermal component has a rather less well defined edge, which, though not conclusive in view of the effect of noise on the lowest contour, suggests that such an envelope could possibly exist. In view of the surface brightness of the easterly extension, the total contribution of this envelope to the flux density of W 49A cannot be more than about $10 \times 10^{-26}$ W m$^{-2}$Hz$^{-1}$ at 1407 MHz. Assuming the spectrum to be flat over the relevant frequency range, this means that $A_{51}$ and $A_{59}$ together now contribute only $16 \times 10^{-26}$ W m$^{-2}$Hz$^{-1}$ at 408 MHz. The flux density of $A_{51}$ is therefore about $11 \times 10^{-26}$ W m$^{-2}$Hz$^{-1}$ at this frequency, which leads to a temperature of 10000°K. If the envelope
had a non-thermal spectrum \((\alpha > 0)\), a lower value of \(T_e\) would be possible, but observations of the source at 178 MHz by Holden and Caswell (1969) show no increase in the flux density of W 49A at low frequencies. Making allowance for the possible errors in the 1407 MHz flux density and for the existence or non-existence of the low brightness envelope it can be concluded only that \(T_e\) lies in the range \(12000 \pm 4000\)°K.

This temperature, together with the free-free flux density, leads to an emission measure of \(1.7 \times 10^6\) pc cm\(^{-6}\) (Equation 3 Chapter IV). The increase in brightness towards the centre of \(A_{51}\) indicates a variation of density in the source, but for simplicity a model with uniform density, and diameter 120" arc will be considered for \(A_{51}\). Such a model would have a density of 450 cm\(^{-3}\) and mass of 4500 M\(_\odot\) of ionised hydrogen. Since \(A_{59}\) is about half the diameter its mass would be about 600 M\(_\odot\) assuming that it has the same density as \(A_{51}\). This assumption is quite reasonable in view of what is known about the flux density of \(A_{59}\). The excitation parameters, defined by \(S_e = R N_e^{2/3}\) where \(R\) is the HII radius and \(N_e\) the electron density of the ionised region, are 250 and 125 pc cm\(^{-2}\) for the two sources. This indicates that both components are being excited by several stars each, as happens in the Orion
nebula. Orion A is, in fact, the only diffuse HII region with a higher emission measure than W 49 (Mezger and Henderson 1967) although some of the high density condensations discussed in the other chapters of this dissertation have considerably higher values. A comparison if its flux density with its angular size indicates that the easterly extension has an optical depth of 0.10 at 408 MHz, assuming that it also has a temperature of 12000°K. Using a cylinder as a model, with the dimensions recorded in Table 4, approximate values of the physical parameters of the extension could be found. These are summarised, together with those for $A_{51}$ and $A_{59}$, in the lower part of Table 4. It can be seen, therefore, that the region contains about 5000 $M_\odot$ altogether, making it one of the very largest HII regions in the Galaxy.

Owing mainly to the different models considered the total mass of gas obtained here is considerably more than in the earlier estimate (Wynn-Williams 1969a). However the conclusions still differ markedly from those of Mezger et al. (1967a) who, from a study of the H109α emission line deduced a temperature of 6300°K and used this to construct a model with a spherical distribution of gas 4' arc in diameter with a density of 234 cm$^{-3}$ and a total mass of 9300 $M_\odot$. 

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The cause of the discrepancy in the electron temperature is not clear, although there are several possible explanations. One is that the results of this paper have shown that most of the HII continuum emission, as well as the OH emission, comes from two fairly distinct regions. If these have different radial velocities, as is reasonable in view of the complexity encountered in the OH spectrum of W 49 (Rogers et al 1967) the 109α line will be broadened and too low an electron temperature will result. Although, with perfect resolution, the effective product of the line width and excess line temperature $T_L \Delta v_L$ would be unaltered by the line being a doublet, visual examination of Figure 1 (10) in the paper by Mezger and Höglund (1967) suggests that an attempt to fit a pair of Gaussian curves to the points available would result in a considerably higher value for the electron temperature. This difficulty was pointed out by Mezger and Höglund in connection with the very low values of electron temperature they derived for W 43 and W 51, sources which have fine structure similar to that in W 49 (Mezger et al. 1967a).

It is also quite likely that there is stimulated emission of the line as discussed by Dyson (1967) and others, but very recently Goldberg (1969) has shown that the low values for electron temperatures of HII regions obtained
using radio recombination line data could be explained by the presence of partially ionised HI clouds around the HII region. Goldberg's theory has not yet appeared in print, but in its application to the Orion nebula he proposed that the optical temperature of about 10000°C could be reconciled with the radio temperature of about 6000°C if there were an intervening layer of neutral gas with an emission measure of the order of 100 - 1000 pc cm\(^{-6}\), about 2% ionised. The ionisation might be by cosmic rays or collisional excitation and, according to Goldberg, such a system would also explain the disagreement with the temperatures determined by the higher order transitions, and the anomalous width for the carbon C109α recombination line. (Palmer et al. 1967; Goldberg and Dupree 1967).

Goldberg's theory is promising for W 49 because, in order to explain the difference in optical depths of the 21 cm HI absorption line profiles from components A and B (Sato et al. 1967), Mezger et al. (1967a) have already proposed that component A is surrounded by a diffuse shell of neutral hydrogen having a density of about 25 cm\(^{-3}\) and a radius of 40 pc. It is possible that the easterly extension visible in Figure 1 is part of such a shell with a higher density or higher degree of ionisation than the rest, and
that Sato's recent results on HI absorption may be explained in terms of an expanding shell of ionised hydrogen around W 49A. However until more details of Goldberg's theory and of its quantitative application to W 49 are available the discrepancy in electron temperature cannot be regarded as resolved. In addition the role of the high density condensations discussed by Mezger et al. (1967a and 1967b) and mentioned in section 5 may well be relevant to this problem.

The positions of the sources of OH emission are shown in Figure 1, using the data of Rogers et al. (1967). The correspondence between the angular positions of the OH sources and the peaks of the HII regions is very close, but it is not possible to say whether the OH emission in W 49 comes from the peak or from one side of the main regions of the continuum, as it tends to do in other OH sources such as W 3, NGC 6334 and DR 21 (Rogers et al 1967; Mezger et al 1967b). The fact that the displacement of the OH sources from the continuum peaks is nearly the same in both magnitude and direction for each component of W 49A, however, suggests that there could be a small consistent error in one of the right ascension determinations and that there might be a closer connection between the centres of OH and HII emission than is indicated by Figure 1.
4. The Non-Thermal Component W 49B

This component is irregular in shape, but within experimental error its centroid lies symmetrically between the main components at both frequencies. It position (1950.0) is:

Right Ascension $19^h\ 08^m\ 43.3 \pm 0.4^s$

Declination $09^\circ\ 00'\ 50'' \pm 20''$

Practically all the emission is confined to an area 4' arc in diameter and any faint structure visible outside this is likely to be spurious. In particular the easterly spur on the 1407 map is probably due to incomplete removal of the grating ring from component A.

From the spectral index in Table 2 and from the spectra published by Mezger et al. (1967a) and more recently by Holden and Caswell (1969), it is evident that the radiation from W 49B is of non-thermal origin. Although obscuration prevents optical confirmation, the spectrum and size (approximately 16 pc at a distance of 14 kpc) of the source are consistent with its being a supernova remnant.

The source at first appears to have a double structure at both frequencies whereas most supernova remnants are observed to have an approximately circular profile. The use of an elongated beam, however, greatly increases the difficulty of interpreting such maps and so the effect of
mapping radially symmetric sources with an elliptical beam was investigated with the aid of the Titan computer (Appendix A).

A range of different brightness distributions $T(r)$ was convolved with a function representing the known response pattern of the telescope at the appropriate declination in an attempt to reproduce the features of the 1407 MHz map (Figure 1). Figure 3 shows one of the most successful attempts, which comprises an isotropically radiating thin shell of Gaussian profile. The brightness temperature $T(r)$ is given by:

$$T(r) \propto \int \exp \left[ -\frac{(r - \sigma)^2}{2\rho^2} \right] dz$$

where $\sigma$ is the radius and $2\rho$ is the thickness of the shell. The integration is along the line of sight and assumes that there is no absorption at this frequency. Figure 3 shows the map that would be produced by the One

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**FIG. 3.** A thin spherical shell of diameter 4' arc as it would appear at 1407 MHz and 9° declination.

**FIG. 4.** Profiles through the centres of W 49B (continuous line) and of the spherical shell model (dashed line) at 1407 MHz.

**FIG. 5.** Profiles through the centres of W 49B at 408 MHz (continuous line) and at 1407 MHz after numerical smoothing (dashed line).

**FIG. 6.** Profiles through the centres of W 49B (continuous line), of the thin spherical shell model (dashed line) and of the circular ring model (dotted line) at 408 MHz.
Nile Telescope at 1407 MHz of such a source at declination 9° with σ = 120° and ρ = 10°.

Although drawn for a specific value of ρ, the map changes very little for ρ over the range 5° - 20°, where the shell thickness is no longer comparable to the source diameter and the telescope beamwidth. The strength of the saddle is raised and the double character became much less pronounced than it is in the real source. Figure 4 shows the profile along the centre line of the source and of the shell model. Apart from the inequality of sizes of the two peaks in the real source and the shell model, there is a general agreement between it and the model confirming that W 49B is a supernova remnant.

Resolution maps of other such sources published at Cas A (Kyle, Kilmare and Kaville 1963) and Tycho's supernova (Baldwin 1967) show that the shells are fairly irregular, with several emission peaks, so that the resolution of W 49B is not unexpected.

This effect, however, is not reflected in the 408 MHz map of W 49B. Figure 5 shows the profile of the source at 408 MHz compared with that at 1407 MHz after numerical smoothing to the same beam size. It is evident that the spectral index on the western side of the source is considerably flatter than on the eastern.
Mile Telescope at 1407 MHz of such a source at declination 9° with $\sigma = 120''$ and $\rho = 10''$.

Although drawn for a specific value of $\rho$, the map changes very little for $\rho$ over the range 5'' - 20'', but above 20'', where the shell thickness is no longer small compared to the source diameter and the telescope beamwidth, the height of the saddle is raised and the double character becomes much less pronounced than it is in the real source. Figure 4 shows the profile along the centre line of the actual source and of the shell model. Apart from the inequality of sizes of the two peaks in the real source, the general agreement between it and the model is good, confirming that W 49B is a supernova remnant. The high resolution maps of other such sources published, such as Cassiopeia A (Ryle, Elsmore and Neville 1965) and Tycho's supernova (Baldwin 1967) show that the shells are fairly irregular, with several emission peaks, so that the asymmetry of W 49B is not unexpected.

This asymmetry, however, is not reflected in the 408 MHz map of W 49B. Figure 5 shows the profile of the source at 408 MHz compared with that at 1407 MHz after numerical smoothing to the same beamwidth. It is evident that the spectral index on the western side of the source is considerably flatter than on the eastern.
Quantitative estimates must be treated with caution as the resolution is low, but by splitting the source into two halves values of 0.6 and 0.45 can be obtained for the eastern and western sides of the source. Since these observations were published (Wynn-Williams 1969a) Hughes and Butler (1969b) have qualitatively confirmed that there is a spectral index variation in W49B over a wide range of frequency: they showed that W49B's mean position moved westwards as measurements at higher frequencies were made. This is in accord with the present results and indicates that the spectral variation is not confined to the range 400 to 1400 MHz.

Although such a variation of structure with frequency is rare in supernova remnants, a somewhat similar behaviour has been noted in IC 443 by Kundu and Velusamy (1968) in which the spectrum of the north east rim was found to be flatter than that of the rest of the source at low frequencies. In both cases the region of low spectral index coincides with the brightest part of the shell at 1400 MHz (Hogg 1964).

In his study of HB3, another supernova remnant near an HII region Caswell (1967) noted that the strongest emission came from the region of the shell nearest the HII region. He also pointed out that the results of Locke et al. (1964) show that the density of both neutral and ionised hydrogen
around IC 443 is also higher near the bright north east rim than elsewhere.

Macdonald (1968) has pointed out that in the supernova remnants W 44 and IC 443 the shell is more completely formed on the side nearer the galactic plane, where the interstellar density is higher, and this behaviour is repeated in W 49B, the galactic plane being to the west of the supernova. However it seems more probable that the enhanced emission on the western side of W 49B is associated with its close proximity to W 49A, especially in view of the existence of the easterly extension from the HII region, which stretches at least half way along the projected separation of the components. The appearance of W 49B is, in fact, quite strong evidence both for the proximity of the two components and for the existence of the low density envelope of neutral hydrogen (Mezger et al. 1967a). The HI absorption results of Sato (1969) are then best explained in terms of local variations of the neutral hydrogen density in the vicinity of W 49A, which is not at all unreasonable in view of what is already known about the complexity of the source.

Van der Laan (1962) has proposed that in some supernova remnants radiation resulting from the compression of the interstellar medium can coexist with that from the original
relativistic electrons. A possible explanation, therefore, of the spectral index variation is that, while on the eastern side the emission is still from the original particles, the increased compression of the interstellar medium on the western side is causing emission with a spectral index appropriate to the flatter energy spectrum of the interstellar medium. It is difficult to estimate the age of the supernova remnant because of the great uncertainty about the interstellar particle density around W 49B but on the basis of its volume emissivity Aizu and Tabara (1967) suggest that its age is of the order of 3000 years. This makes it intermediate in age between young sources such as Cassiopeia A and Tycho's supernova, and old remnants such as IC 443 and the Cygnus Loop, so that in W 49B one might well expect emission from both the original particles and the compressed interstellar medium.

A second interesting feature of the 408 MHz profile of W 49B is that the depression in the middle is much greater than would be expected for a uniformly radiating spherical shell source, as can be seen in Figure 6. For comparison, the third, dotted, line in Figure 6 is the profile that would result at 408 MHz from a source consisting of a thin circular ring of emission, its axis parallel to the line of sight, with no radiation from the central region.
It can be seen that W 49B appears to be intermediate between the two models. There are several factors that might contribute to this effect.

(a) Absorption inside the supernova shell. This effect would be stronger at low frequencies, as observed, but for the emission measure to be large enough to cause significant absorption at 408 MHz, the thermal emission from the absorbing medium would be greater than the non-thermal emission from the shell above about 10 GHz. However the results of Mezger et al (1967a) show that at 15.4 GHz the flux density of W 49B is still compatible with a non-thermal spectrum. In addition, Mezger and Höglund (1967) reported the absence of the Hα recombination line in W 49B, which indicates that any thermal emission must be very weak.

(b) Irregularities in the shell. To obtain the profile observed at 408 MHz one would have to postulate that the shell had two strong peaks of emission, one on each side of the source. This is reasonable on the western side, since Figure 4, the 1407 MHz profile, also shows some signs of such a peak, but on the eastern side the spectral index would have to be very steep indeed to explain both the profiles.
(c) Directed emission in the shell. A profile such as that in Figure 6 might arise if the synchrotron radiation were enhanced in directions tangential to the shell's surface. The existence of such directed emission has been postulated by Baldwin (1967) to occur in Tycho's supernova, but the absence of the effect at 1407 MHz in W 49B makes the explanation less attractive. Observations at different polarisations would be useful here.

It is not clear whether the reduced emission from the centre of the shell is associated with the variation of spectral index, or should be regarded as a separate phenomenon. High resolution studies of other such sources of different ages are very important in ascertaining to what extent these effects are common to all supernova remnants.

5. W 49A at 5 GHz.

From its spectrum Mézger et al. (1967a) concluded that W 49A contained one or more high density HII condensations with a turnover frequency of about 6 GHz. These condensations have a total free-free flux density of about $27 \times 10^{-26} \, \text{W m}^{-2} \text{Hz}^{-1}$ but contribute negligible radiation at 408 and 1407 MHz. Since these condensations are of
great importance to the physics of W 49 and to the study of HII regions in general, one of the first experiments with the 5 GHz system on the One Mile Telescope was to look for structure in W 49A on the scale of a few seconds of arc.

Only two positions of the moving aerial were used in this pilot survey, so the map obtained of W 49A is exceedingly confused by grating rings overlapping from different parts of the source. The situation is made even worse by the fact that when the observations were made the phase stability of the One Mile Telescope at 5 GHz was uncertain owing to thermal drifts in the rather long lengths of polythene filled coaxial cable then in use in the system.

However the data obtained are valuable as they confirm that there is a good deal of structure in W 49A on a scale 5" - 20" arc. The basically double nature of the source \(A_{51} + A_{59}\) is also clearly visible on the map but because of the grating rings it is not possible to obtain any new positions for the components or estimate the number or the sizes of the condensations.

The prospects of making a useful, more extended, survey of W 49A at 5 GHz are quite good. A 16-position (32-spacing) synthesis could map components \(A_{51}\) and \(A_{59}\)
together without their interfering with each other. To include the easterly extension, which is of rather less interest, would require double the observation time, which, in any case, is not possible owing to the limited number of localities of the moving aerial which have 5 GHz local oscillator connections. Because of its non-thermal spectrum W 49B is considerably weaker than W 49A at 5 GHz and, by aiming the aerials at a point mid-way between $A_{51}$ and $A_{59}$, component B can be made to lie in the first minimum of the envelope polar response of the 18m paraboloids. Interference by grating rings from W 49B can thus be reduced to an acceptable level without resorting to complicated computational techniques. It turns out that W 49 has the unusual property that to produce a map free of grating rings requires more observation time at 2.7 GHz than at 5 GHz.

It is hoped to commence the high frequency study of W 49A fairly soon so that the positions, sizes and flux densities of the condensations can be found. As well as obtaining valuable data about the evolution of compact HII regions (Chapter VI) it will be very interesting to attempt to correlate the positions of the condensations with the regions of molecular activity for which W49 has become notorious in the last few months.
III. THE COMPACT HII REGION DR 21

1. Introduction.

DR 21 is part of the diffuse HII region W 75 and is the most intense of the thirty or so sources listed by Downes and Rinehart (1966) in their 5GHz survey of the Cygnus X region. It is 3' arc south of an intense source of OH emission (Palmer and Zuckerman, 1967; Raimond and Eliasson, 1969) near to which formaldehyde absorption and water emission have also been detected (Snyder et al. 1969; Cheung et al. 1969). The continuum source was mapped by Ryle and Downes (1967) with the One Mile Telescope at 408 and 1407 MHz and was shown by them to have a very high turnover frequency and, consequently, a very large emission measure. DR 21 was, in fact, the first of the high density HII condensations discussed by Mezger et al. (1967b) to be discovered and has the largest angular size of those known. According to Raimond and Eliasson (1969) and Hyland (1969) infra-red emission has been detected near DR 21 but the position of these sources has not yet become available. The One Mile Telescope has now been used to map DR 21 at 2695 and 4995 MHz and this chapter is concerned with the results of these observations.
2. Observations.

Owing to technical difficulties operation of the One Mile Telescope at both frequencies simultaneously has not been possible to date. The 5 GHz observations (Fig. 7) were made in the summer of 1968 and those at 2.7 GHz (Fig. 8) about three months later: in both cases 4 positions of the moving aerial were used, producing an 8-spacing map. The maps are presented in aerial coordinates, with the declination scale compressed by a factor 0.671 (sin 42°).

Ryle and Downes (1967) showed that the 408 and 1407 MHz data can be explained in terms of a model consisting of three components which they designated A, B and C. At the frequencies at which the present observations were made component C is much too dim to be seen and almost all the radiation is emitted from component A, which has a turnover frequency of about 3 GHz and an optically thin flux density of about \(19 \times 10^{-26} \text{ W m}^{-2} \text{Hz}^{-1}\). The low brightness easterly extension visible in Fig. 8 corresponds to component B, which has a turnover frequency of 600 MHz and an optically thin flux density of \(0.8 \times 10^{-26} \text{ W m}^{-2} \text{Hz}^{-1}\). Although not very evident in Fig. 7 a similar extension is clearly visible in a more finely contoured version of the 5 GHz map.

Component A, with which all the rest of this chapter
FIG. 7. DR 21 at 4995 MHz (aerial coordinates). The contour interval is 223°K. The shaded circle shows the half-power beamwidth and the arms of the 'L' motif are 10'' arc long in both coordinates.
in concerned, is well resolved at 5 GHz and has quite a complicated structure. The position of its brightness peak at this frequency is significantly displaced from 1407 MHz, as can be seen from Table 5. This is just as one would expect from the fact that the southern part of the source appears brighter than the northern part at 5 GHz. Since the southern part of the source has a higher emission measure than the northern it will begin to show the effect of self absorption at a corresponding lower frequency. This will mean that the southern part will contribute proportionally less flux than the northern part, just as is observed.

2. The Electron Temperature of DR 21

3. The Electron Temperature of DR 21

FIG. 8. DR 21 at 2695 MHz (aerial coordinates). The contour interval is 372°K. The shaded circle shows the half-power beamwidth and the arms of the 'L' motif are 10" arc long in both coordinates.
is concerned, is well resolved at 5 GHz and has quite a complicated structure. The position of its brightness peak at this frequency is significantly displaced from that at 1407 MHz, as can be seen from Table 5. This is just as one would expect from the fact that the southern part of the source appears brighter than the northern at 5 GHz. Since the southern part of the source has a higher emission measure than the northern it will begin to show the effect of self absorption at a correspondingly higher frequency. This will mean that the southern part of the source will contribute proportionately less flux at 1407 MHz, and so the mean position of the whole source will move northwards at low frequency, just as is observed.

3. The Electron Temperature of DR 21

Mezger et al. (1967b) observed the 5 GHz H109α recombination line in DR 21 and deduced an electron temperature of 7600°± 350°K for the source. This is somewhat below the value of 11 000 °K obtained by Ryle and Downes (1967) based on an estimated 20" arc Gaussian diameter for component A at 1407 MHz.

As discussed in Chapter II and Appendix A the measured Gaussian diameter of an HII region is generally an underestimate of its actual size, so that the electron temperature
<table>
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<th>GHz</th>
<th>$10^{-26}$ W m$^{-2}$ Hz$^{-1}$</th>
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<tr>
<td>0.408</td>
<td>0.75 ± 0.08</td>
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<tr>
<td>1.407</td>
<td>6.4 ± 0.6</td>
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<tr>
<td>2.695</td>
<td>12.5 ± 1.5</td>
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<td>4.995</td>
<td>18 ± 2</td>
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<td>8.25</td>
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<td>91.0</td>
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Table 5. Flux densities and positions of the brightness peak of DR 21. The data are from Ryle and Downes (1967), Riegel and Epstein (1968) and the present work. They are denoted by RD, RE and WW respectively.
calculated from such a parameter may well be too high. Indeed, Fig. 7 shows that the diameter of DR 21 when optically thick must be near 30'' arc, a value close to that proposed by Mezger et al. (1967b) based on H109α line measurements. To obtain a more accurate estimate of the electron temperature the method described by Terzian et al. (1968) was applied to DR 21. This involves assuming a uniform electron temperature for the source and using the measured brightness temperature distribution at 5 GHz (Fig. 7) to calculate the emission measures at a series of points spaced somewhat less than half a beamwidth apart all over the source. The optical depths were then calculated for other, lower, frequencies, and the total predicted flux density compared with the experimental values. Fig. 9 shows that a good fit to the observed points could be obtained for $T_e = 6000^\circ K$, a value significantly lower than obtained in both previous determinations. This method is, however, liable to lead to underestimation unless the HII region is very much larger than the beamsize. The reason for this is that at sufficiently low frequency the flux density predicted for the source will be equal to that from an optically thick source subtending a solid angle equal to that of the lowest non-zero contour on the 5 GHz map. Because of the broadening effect of using a finite beamwidth this solid angle will
4. The Physical Conditions in DR 21

FIG. 9. Measured flux densities of DR 21 (component A) compared with calculated spectra for electron temperatures of 6000°K and 10000°K.
always be larger than the source itself, resulting in an electron temperature which is too small. To gauge this effect the procedure was repeated on DR 21 using the 5 GHz map after it had been convolved with a suitable function so as to give it the same resolution as the 2.7 GHz map. The broadening effect is worse this time of course, and the best agreement was obtained for $T_e = 4000{^\circ}K$. Terzian et al. (1968) obtained temperatures for the Orion Nebula and NGC 2024 by this method which were only about half those obtained from radio recombination line data. It is thus almost certain that the temperature of DR 21 is above 6000{^\circ}K. There is no evidence to contradict Mezger et al.'s estimate of 7600{^\circ}K, but the temperature could easily be higher than this. There does not seem to be any discrepancy as drastic as that which has arisen in the case of W 49, anyway.

4. The Physical Conditions in DR 21

In order to compare DR 21 with other compact HII regions whose shapes are unknown it is useful to fit its data to a simple model. Models comprising a sphere of uniform density and a Gaussian density distribution have been proposed by Mezger et al. (1967b) and by Hughes and Butler (1969a). As discussed by Hughes and Butler and also in chapter V the uniform density model will have a spectrum with a significantly
sharper turnover than the Gaussian, but the spectrum of DR 21, though not as well observed as that of NGC 7027, seems to be intermediate between those of the two models. Fig. 7 suggests that there are quite extensive density variations in the source but since there are also signs of a sharp boundary on the southern side the parameters of a uniform density spherical model with $T_e = 7600^\circ K$ have been derived as a first approximation. Because the turnover is not as sharp as in the model the turnover frequency is not easy to define accurately. A value of 3 GHz has been adopted from which an emission measure of $1.5 \times 10^7$ pc cm$^{-6}$ may be derived.

The distance to DR 21 is uncertain. The Cygnus X group of sources are thought to be about 1.5 kpc from the Sun, but, on account of its obscuration, Ryle and Downes (1967) suggested that DR 21 is situated at a rather greater distance, perhaps in the Perseus arm at 6 kpc. More recent studies of HI absorption (Sato 1969) and OH emission (Winnberg 1969) have indicated that the distance to DR 21 is probably nearer 1.5 kpc, and this distance will be adopted for the remainder of this chapter. The calculated physical parameters are listed in Table 6. The final column indicates the dependence of each quantity on the assumed distance to the source.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electron Temperature</td>
<td>7600°K</td>
<td>(-)</td>
</tr>
<tr>
<td>Distance (d)</td>
<td>1.5 kpc.</td>
<td>(-)</td>
</tr>
<tr>
<td>Emission Measure</td>
<td>1.5 × 10^7 pc cm^{-6}</td>
<td>(-)</td>
</tr>
<tr>
<td>Diameter (2R_o)</td>
<td>0.22 pc</td>
<td>(d)</td>
</tr>
<tr>
<td>Electron Density (N_e)</td>
<td>8500 cm^{-3}</td>
<td>(d^{-0.5})</td>
</tr>
<tr>
<td>Mass of Ionised Hydrogen</td>
<td>1.6 M⊙</td>
<td>(d^{2.5})</td>
</tr>
<tr>
<td>Excitation Parameter (S_o)</td>
<td>46 pc cm^{-2}</td>
<td>(d^{0.67})</td>
</tr>
</tbody>
</table>

Table 6. Adopted physical parameters of DR21. The dependence of each quantity on the assumed distance is indicated in the last column. The excitation parameter is defined by the equation:

\[ S_o = R_o N_e^{2/3} \]
The conclusions arrived at in this chapter are very similar to those of Mezger et al. (1967b) and Mezger (1968) and lend support to the hypothesis that DR 21 is an example of what Davidson and Harwit (1967) call a "cocoon" star, with the properties outlined in Chapter I. The fact that the diameter of the source is seen to be very close to that expected on the basis of thermal emission mechanisms is very strong evidence against the "pre-protostar" theory of Hughes (1969). Hughes proposed that the radiation from the compact sources in W 49, W 3 and DR 21 is non-thermal in origin and that the turnover in the spectrum is due to synchrotron self absorption in a collapsing protostar. Making assumptions about the particle energy spectrum and the magnetic field in a protostar Hughes concluded that such objects with diameters in the range 1 - 100 A.U. could explain the radio frequency emission from the compact sources. It can be seen from Table 6 that DR 21 is, in fact, very much larger than this. To reconcile the very flat high frequency spectrum of DR21, measured to 3mm wavelength, with the steeper spectrum associated with synchrotron radiation Hughes' theory requires the coexistence of several protostars with suitably differing turnover frequencies, but, except for the sharp southern edge, DR 21 now seems fully resolved and does not resemble a collection of point sources at 5 GHz. The HII
condensations hypothesis would thus seem far more satisfactory, both in providing agreement with observed data and in requiring fewer assumptions of poorly known quantities, such as particle energy spectra and magnetic fields in protostars. The evolution of HII condensations, including DR 21, is discussed in Chapter VI.

In their radio survey at 2600 MHz Thompson and Colvin (1967) found that N83 8037 appeared to have a much larger flux density than any other planetary nebula observed, although they pointed out that the source was probably confused. Its radio brightness was surprising as, optically, the object is not very prominent and has not been greatly studied. Until the present observations no attempt has been made to map the source, because the optical diameter of the nebula is below the resolution limit of other radio telescopes. OH emission has been detected from the nebula (Elliot et al., 1967) but this discovery did not appear in print until after the present data had been analysed and a paper on the subject submitted for publication (Wynn-Williams, 1969). The contents of Sections 2 - 4 is very similar to that of the paper, but knowledge of the distance to the source, derived from OH data, has permitted tentative estimates of the sizes and masses of the components to be made in Section 5. Discussion of the evolution of the sources is, again, left until Chapter VI.
IV. THE GALACTIC NEBULA NGC 6857

1. Introduction.

NGC 6857 was classified by Seyfert (1947) as a fairly small approximately circular planetary nebula. In their radio survey at 2840 MHz Thompson and Colvin (1967) found that NGC 6857 appeared to have a much larger flux density than any other planetaries observed, although they pointed out that the source was probably confused. Its radio brightness was surprising as, optically, the object is not very prominent and has not been greatly studied. Until the present observations no attempt had been made to map the source, because the optical diameter of the nebula is below the resolution limit of other radio telescopes. OH emission has been detected from the source (Elldér et al. 1969) but this discovery did not appear in print until after the present data had been analysed and a paper on the subject submitted for publication (Wynn-Williams 1969b). The contents of Sections 2 - 4 is very similar to that of the paper, but knowledge of the distance to the source, derived from OH data has permitted tentative estimates of the sizes and masses of the components to be made in section 5. Discussion of the evolution of the sources is, again, left until Chapter VI.
2. Observations.

Observations were made in the Autumn of 1968 at the single frequency of 2695 MHz, at which the half-power beam-widths of the One Mile Telescope are 12" arc in Right Ascension and 22" arc in declination. Since the large angular size of the source was not anticipated only four positions of the moving aerial were used, with the result that the separation of the brightest components of the source was almost equal to the radius of the primary grating ring around each of them. The final map (Fig. 10) is therefore a combination of the two synthesised maps that were computed using the source removal technique of Neville et al. (1969) on components A and C in turn. Although this technique can result in a slight increase in the noise level in the vicinity of the removed rings, the additional information about the source that might be obtained from more extended observations was not considered worth the 48 hours of extra observing time that would be required.

Emission is now clearly discernable from four regions, a component of fairly low brightness corresponding to the

---

FIG. 10. The region of sky around NGC 6857 at 2695 MHz in sky coordinates. The optical position of NGC 6857 is marked by a dotted circle and of K 3-50 by a cross. The contour interval is 79°K and the half-power beamwidth is represented by the shaded ellipse. The arrow is in a direction parallel to the Galactic plane (See Chapter VI, sec 4).
NGC 6857

2695 MHz

The effective spectra of NGC 6857, including the compact sources H30 and C. The letter refers to the observations listed in Table 1.

FIG. 10.
optical position of NGC 6857, and three other compact sources lying to the north of the nebula which have been designated A, B and C in order of increasing right ascension. The measured parameters of the four components are listed in Table 7. Due to its lower surface brightness accurate estimations of the radio size and flux density of NGC 6857 itself are difficult, partly because of the presence of background nebulosity and partly because of the probable distortion of its shape due to imperfect removal of the grating ring from component C. The size of the radio emitting region is, however, compatible with the 40" arc diameter of the optical nebula (Perek and Kohoutek 1967) and, except for the flux density, the data in Table 7 corresponding to this component refer to the optical nebula. Component A, which is unresolved and has a peak brightness temperature of over 1000 K, lies less than 3" arc away from the small optical nebula K 3-50 in Perek and Kohoutek's catalogue and is thus almost certainly associated with it. The Sky Survey prints of this region are very crowded and no reliable identifications can be suggested for components B and C (Plate I).

FIG. 11.

The effective spectrum of NGC 6857 including the compact sources A, B and C. The letters refer to the observations listed in Table 8.
NGC 6857

Table 7. Measured parameters of the four components. Those marked - are taken from Bayfort (1947) rather than from the radio data.

FIG. 11.
<table>
<thead>
<tr>
<th></th>
<th>Position (1950.0)</th>
<th>Sizes (R.A. x Dec.)</th>
<th>Flux Density (10⁻²⁶ W m⁻² Hz⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Measured</td>
<td>Gaussian</td>
</tr>
<tr>
<td>A</td>
<td>19^h 59^m 49.95 ± 0.1^s</td>
<td>&lt;13'' x &lt;24''</td>
<td>&lt;5'' x &lt;9''</td>
</tr>
<tr>
<td>B</td>
<td>19^h 59^m 52.3 ± 0.5^s</td>
<td>23'' x 30''</td>
<td>20'' x 22''</td>
</tr>
<tr>
<td>C</td>
<td>19^h 59^m 58.6 ± 0.2^s</td>
<td>21'' x 26''</td>
<td>17'' x 13''</td>
</tr>
<tr>
<td>NGC 6857</td>
<td>19^h 59^m 52^s *</td>
<td>40'' *</td>
<td></td>
</tr>
</tbody>
</table>

Table 7. Measured parameters of the four components. Those marked * are taken from Seyfert (1947) rather than from the radio data.
Table 8. Flux densities attributed to NGC 6857 including components A, B and C. The final column refers to the abbreviations in Fig. 11.

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Flux Density (10^{-26} \text{ W m}^{-2}\text{Hz}^{-1})</th>
<th>Observer</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>178</td>
<td>(&lt; 3)</td>
<td>Pilkington and Scott (1965)</td>
<td>4C</td>
</tr>
<tr>
<td>408</td>
<td>(4.0 \pm 0.5)</td>
<td>Ficarra and Pardielli (1968)</td>
<td>FP</td>
</tr>
<tr>
<td>430</td>
<td>(1.2 \pm 0.2)</td>
<td>Terzian (1966)</td>
<td>Te</td>
</tr>
<tr>
<td>1400</td>
<td>(5.0 \pm 0.4)</td>
<td>Thompson (1968)</td>
<td>Th</td>
</tr>
<tr>
<td>2695</td>
<td>(7.8 \pm 1.0)</td>
<td>Wynn-Williams (1969b)</td>
<td>WW</td>
</tr>
<tr>
<td>2840</td>
<td>(6.4 \pm 0.5)</td>
<td>Thompson and Colvin (1967)</td>
<td>TC</td>
</tr>
<tr>
<td>7400</td>
<td>(10.4)</td>
<td>Davies (1968)</td>
<td>D</td>
</tr>
</tbody>
</table>

Although no direct information is available about the spectra of the individual components, it is clear from Fig. 11 that the overall emission from the source has a thermal spectrum, although one of the weaker components could conservatively have some non-thermal emission. The fact that the flux density at 178 MHz is less than \(3 \times 10^{-26} \text{ W m}^{-2}\text{Hz}^{-1}\),...
3. Spectra of the Components

Determinations by various observers of the flux densities attributed to NGC 6857, which in fact represent the sum of the flux densities of all four sources in the area are listed in Table 8 and shown in Fig. 11. The four components are superimposed on a 22' arc diameter thermal radio source W58 (NRAO 621) which contributes about $50 \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$ at frequencies above 750 MHz (Pauliny-Toth et al. 1966), thus making measurements with a single dish rather difficult. For this reason Ficarra and Padrielli's determination using the Northern Cross Telescope is considered more reliable than that of Terzian at a similar frequency as their resolving power was considerably greater. NGC 6857 is in a region of sky in which the 4C catalogue may be incomplete because of the presence of sidelobes from Cygnus A (Pilkington and Scott 1965). Re-examination of the 4C records, however, places an upper limit of $3 \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$ on the total emission from the four components at 178 MHz.

Although no direct information is available about the spectra of the individual components, it is clear from Fig. 11 that the overall emission from the source has a thermal spectrum, although one of the weaker components could conceivably have some non-thermal emission. The fact that the flux density at 178 MHz is less than $3 \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$,
however, means that such a component would have to have a spectral index lower than 0.3 whereas very few extended extragalactic radio sources have spectral indices of less than 0.5 over the range of frequency in question.


It can thus be concluded that at least three and probably all four of the components are of thermal origin with differing turnover frequencies, and would therefore seem to be a group of either planetary nebulae, HII condensations or conceivably a mixture of the two. Planetary nebulae have a typical population II disk distribution and close associations such as that of NGC 6857 and K 3-50, which are only 1.2' arc apart are not common. Only two other pairs (K 2-9/M 2-37, which are separated by 1.4' arc and K 4-44/NGC 7008, separated by 0.4' arc) in Perek and Kohoutek's catalogue have separations of less than 3' arc. There is certainly no optical evidence for the existence of clusters of three or more planetaries with diameters of a few minutes of arc. The distance of NGC 6857 has been estimated by Perek (1963) using Shklovsky's method (1956) as 900 pc. If the quartet of Figure 10 were an association of planetaries at this distance, the projected separation of A and C would be only 0.6 pc.

However NGC 6857 is overlapped by the diffuse nebula
Sharpless 100 (Sharpless 1959) which has a diameter of 4' arc and it has been pointed out by Minkowski (1967) that it much more closely resembles a bright HII condensation than a planetary nebula. The spectra have not been published for the exciting stars of either NGC 6857 or K 3-50 so the possibility remains that the excitation in this region is by young OB stars, which generally occur in clusters, rather than the highly evolved stars usually associated with planetary nebulae.

For any optically thin radio source the ratio of radio to optical brightness is only a slowly varying function of temperature. All four components in Figure 10 have comparable flux densities, but on the Sky Survey prints (Plate I) K 3-50 is at least four magnitudes fainter than NGC 6857, and, as stated above, components B and C are not associated with any bright nebulosity. Therefore if the four objects represent a real association in space they must be accompanied by a large quantity of obscuring matter. This is much more likely if the exciting stars are young, population I objects. In this context it is interesting to note that the preliminary photometric measurements of Liller and Shao (1968) indicate an unusually large degree of reddening in the light from the central star of NGC 6857, compared with that from the stars of most other planetaries. It is not clear how much of the
reddening is to be attributed to the nebular shell and how much to the interstellar medium, but Reddish (1967, 1968) has shown that O and B stars in young clusters frequently display considerable reddening which is due almost entirely to circumstellar dust clouds.

The emission measure of component A is greater than $10^7$ pc cm$^{-6}$, which, though not abnormal for planetary nebulae, is unusually high for diffuse HII regions. The characteristics of component A are, however, in many ways rather similar to those of the high density condensations whose existence in the diffuse HII regions W 3, W 49 and DR 21 has been postulated by Mezger et al. (1967) and Mezger (1968), and with which this dissertation is largely concerned. If component A were such a condensation embedded in the diffuse HII region W 58, both its high emission measure and large optical extinction would be explicable. It is likely that components B, C and NGC 6857 itself are similar condensations in a later stage of evolution, and this idea is developed further in section 5 and in Chapter VI.

The conclusion that the four objects are associated with a region of star formation is strongly supported by the discovery of an intense source of OH emission at position $19^h 59^m 51^s \pm 15^s$, $33^\circ 25' \pm 3'$ (Elldér et al 1969) at
a frequency which corresponds to a kinematic distance of 8.2 kpc. The positional accuracy of the OH measurement does not allow identification with any of the condensations in particular, as all four components lie within the error ellipse of the OH position. In any case most OH sources do not coincide with significant optical or radio objects. However many of the Galactic OH emission sources which have been identified with optical or continuum radio sources are associated with regions of star formation, whereas no OH emission has been recorded from a planetary nebula. It is significant that a common feature of the condensations discussed by Mezger et al. (1967b) is that all coincide with or are close to regions of OH emission, although, admittedly, there is a very strong selection effect operating here.

It would be instructive to look for infra-red radiation from heated dust in the source and to measure its hydrogen recombination lines, but most useful would be more optical data about the spectra and colours of the associated stars in order to assign them more confidently to population I or II. If it turns out that NGC 6857 and K 3-50 are planetary nebulae after all we must conclude that the association of four bright thermal radio sources is a coincidence or that clustering of these late-type objects is possible. Because this is rather unlikely, in the next section it will
be assumed that all four sources are being excited by young stars and that none of them is a planetary nebula.

5. Sizes and Masses of the Components.

In this section the sizes and densities of the components are calculated, based on Eldére et al's estimate of the distance to the OH source in W 58. The results are rather tentative since the distance estimate is subject to a great deal of uncertainty (Winnberg 1969). However none other is available.

Each condensation is represented, slightly arbitrarily, by a model comprising an isothermal sphere of gas of uniform density. In the case of components B, C and NGC 6857 itself the size of the sphere is inferred directly from the map, while for component A additional assumptions about the spectrum of the source are necessary. For all components an electron temperature of 8000°K has been assumed. As well as being close to the measured temperature of DR 21, this value allows direct comparisons to be made with the Gaussian density distribution models of Hughes and Butler (1969a). In general the dependence of the physical parameters on electron temperature is not strong and, in any case, is indicated in Tables 9 and 10.

A modelfitting procedure (Appendix A) was used to
determine the diameters of components B and C, while for
NGC 6857 the optical diameter of Seyfert (1947) was used.
The central brightness temperatures of all three sources
indicates that they are optically thin at 2.7 GHz, so the
optical depth (τ) through the centre of the source can be
calculated using Mezger and Henderson's relation (1967):

\[ S_\nu = \frac{2kT_e}{c^2} \frac{\nu^2}{\pi \theta^2} \tau \]  

(1)

where θ is the angular diameter of the source.
The emission measure (EM) is found from the approximate
expression:

\[ \tau = 0.3 \left(\frac{EM}{pc \ cm^{-6}}\right) \left(\frac{\nu}{MHz}\right)^{-2} \left(\frac{T_e}{10^4 \degree K}\right)^{-3/2} \]  

(2)

The optically thin flux density (S_{ff}) is then independent
of frequency and is given by:

\[ \left(\frac{S_{ff}}{Wm^{-2}Hz^{-1}}\right) = 1.13 \times 10^{-9} \left(\frac{EM}{pc \ cm^{-6}}\right) \left(\frac{\theta}{\text{arcsec}}\right)^2 \left(\frac{T_e}{10^4 \degree K}\right)^{-7/2} \]  

(3)

From the emission measure the density, mass and
excitation parameter (S_e) are easily calculated and are listed
in Table 9. The dependence of each parameter on d, the
assumed distance to the source and T_e, the electron temperature,
are also indicated. The turnover frequency, at which \( \tau = 1 \),
is obtained from equation 2. The fact that all three sources
have turnover frequencies in the range 0.4 - 2 GHz is
encouraging since the spectrum of all the components together is then more likely to resemble Figure 11.

From the flux density at 7.4 GHz it must be concluded that component A is still optically thick at 2.7 GHz, and has a flux density of $4.3 \times 10^{-26}$ W m$^{-2}$Hz$^{-1}$ at 7.4 GHz. This value must be close to the optically thin flux density since if A were still optically thick at 7.4 GHz its flux density would be about eight times higher than at 2.7 GHz which is contrary to observations. The turnover frequency of A is therefore probably about 5 GHz. From equations 2 and 3 the emission measure, angular diameter and all the other parameters in Table 10 may be derived.

Because of the lack of individual spectral information the results for all four sources are very preliminary. They do, however provide a powerful insight into a possible mode of evolution of clouds surrounding young stars. This is discussed in Chapter VI.
<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th>C</th>
<th>NGC6857</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flux Density (S_{ff})</strong></td>
<td>1.4</td>
<td>3.6</td>
<td>1.3</td>
</tr>
<tr>
<td>(10^{-26} \text{ W m}^{-2}\text{Hz}^{-1})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Angular Diameter ((\theta))</strong></td>
<td>33''</td>
<td>25''</td>
<td>40''</td>
</tr>
<tr>
<td>(arc sec)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Diameter</strong></td>
<td>1.32</td>
<td>1.00</td>
<td>1.60</td>
</tr>
<tr>
<td>(pc)</td>
<td></td>
<td></td>
<td>(d)</td>
</tr>
<tr>
<td><strong>Emission Measure (EM)</strong></td>
<td>0.99</td>
<td>4.4</td>
<td>0.62</td>
</tr>
<tr>
<td>(10^6 \text{ pc cm}^{-6})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Electron Density (N_e)</strong></td>
<td>865</td>
<td>2100</td>
<td>620</td>
</tr>
<tr>
<td>(\text{cm}^{-3})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Excitation Parameter (S_o)</strong></td>
<td>60</td>
<td>82</td>
<td>58</td>
</tr>
<tr>
<td>(\text{pc cm}^{-2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mass of Ionised Hydrogen (M_\odot)</strong></td>
<td>34</td>
<td>36</td>
<td>44</td>
</tr>
<tr>
<td><strong>Turnover Frequency</strong></td>
<td>600</td>
<td>1150</td>
<td>430</td>
</tr>
<tr>
<td>(MHz)</td>
<td></td>
<td></td>
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<table>
<thead>
<tr>
<th></th>
<th>(d^{1/2})</th>
<th>(T_e^{1/4})</th>
<th>(d^{-1/2}T_e^{1/4})</th>
<th>(d^{1/2}T_e^{1/6})</th>
<th>(d^{5/2}T_e^{1/4})</th>
<th>(T_e^{-1})</th>
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</thead>
<tbody>
<tr>
<td><strong>Physical parameters of the larger components associated with NGC 6857</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 10. Physical parameters of component A (K 3–50).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux Density ($S_{ff}$)</td>
<td>$4.3 \times 10^{-26}$ W m$^{-2}$Hz$^{-1}$</td>
</tr>
<tr>
<td>Angular Diameter ($\Theta$)</td>
<td>6&quot; arc</td>
</tr>
<tr>
<td>Diameter</td>
<td>0.24 pc</td>
</tr>
<tr>
<td>Emission Measure</td>
<td>$8.3 \times 10^7$ pc cm$^{-6}$</td>
</tr>
<tr>
<td>Electron Density</td>
<td>18500 cm$^{-3}$</td>
</tr>
<tr>
<td>Excitation Parameter</td>
<td>84 pc cm$^{-2}$</td>
</tr>
<tr>
<td>Mass of Ionised Hydrogen</td>
<td>3.9 $M_\odot$</td>
</tr>
<tr>
<td>Turnover Frequency (assumed)</td>
<td>5000 MHz</td>
</tr>
</tbody>
</table>
1. Introduction.

Since many of the physical parameters of the compact HII condensations described in the earlier chapters have much resemblance to those of dense planetary nebulae it was decided to extend observations to include NGC 7027, the planetary with the highest known emission measure. This nebula is convenient for comparison purposes since it has a turnover frequency ($\sim 4$ GHz) close to that of DR 21 and is also bright enough for Mezger et al. (1967b) to have measured its 5 GHz $\text{H$_2$O}$ recombinaiton line and obtained an estimate of the electron temperature by this method.

NGC 7027 is also an extremely interesting object in its own right. The elusiveness of its central star has been remarked on many times: the fact that it has never been detected has led Aller et al. (1955) to conclude that it must have a surface temperature well in excess of $150,000^\circ$K, making it one of the hottest stars known.

The nebula has a very rich optical spectrum which has been extensively analysed by Aller (1954) and others, and a remarkable infra-red excess (Gillett et al. 1967): at $20\mu$ its flux density is two orders of magnitude greater than would be expected from thermal bremsstrahlung alone. The nebula
has an irregular shape and both direct photographs (Minkowski 1968) and spectral evidence (Aller 1954) indicate that it has a filamentary character. Detailed studies of the morphology of the nebula are hampered by its small angular size, and, until now, it has not been possible to map the source at radio wavelengths. However at 5 GHz the beam of the One Mile Telescope is narrow enough to resolve NGC 7027 and this chapter is concerned with the results obtained from the recent observations of the source at this frequency.

2. Observations.

Figure 12 shows the map obtained of NGC 7027 with the One Mile Telescope at 5 GHz, at which frequency the half power beamwidths are 6.5" arc in right ascension and 8.7" arc in declination. The map is the result of a 4 - spacing (2-position) synthesis computed in the usual way (Chapter I). The map is in aerial coordinates, so the source appears foreshortened in declination. The AGK2 position of the nebula is marked with a cross and is identical with that given by Kohoutek in Perek and Kohoutek's catalogue (1967). The latter authors quote a positional accuracy of 0.1" arc for this nebula. Since the radio position at this frequency is accurate to about 0.5" arc there is a significant displacement of the radio
NGC 7027
4995 MHz

30"

42° 02' 00"

10^5 21^h 05^m 08^s

FIG. 12. NGC 7027 at 4995 MHz (aerial coordinates). The contour interval is 650°K and the cross represents the AGK2 position. The shaded circle shows the half-power beamwidth and the 'L' motif the foreshortening.
object 1.5" arc to the east of the optical. This discrepancy is discussed in section 3.

The source is extended along a line at P.A. 120° ± 20° on the sky by an amount that corresponds to a Gaussian diameter of 6" arc. It is unresolved along a line normal to this axis, which means that its Gaussian diameter is less than 5" arc in this direction. The nebula has a peak brightness temperature of 4800°K and an integrated flux density of 6.5 ± 0.6 x 10^{-26} W m^{-2} Hz^{-1}, in good agreement with other determinations at around this frequency. (Table 11 and Figure 13).

3. Comparison with Optical Data.

Plate II is a reproduction of the series of photographs taken by Minkowski (1968) of NGC 7027 in Hα light with the 200" telescope. These photographs were published with no indication of scale or orientation, but an estimate of these is possible from Curtis's illustration (1918), which is derived from a composite of several exposures with the Lick Crossley reflector. Curtis describes the nebula as follows: "The condensations are not stellar in the shortest exposures. Quite irregular and roughly trinuclear, though the southern condensation is apparently composed of two masses close together. The southern condensation and
<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Flux Density $\left(10^{-26} \text{Wm}^{-2}\text{Hz}^{-1}\right)$</th>
<th>Observer</th>
</tr>
</thead>
<tbody>
<tr>
<td>408</td>
<td>$&lt; 0.1$</td>
<td>Ficarra and Padrielli 1968</td>
</tr>
<tr>
<td>408</td>
<td>$&lt; 0.2$</td>
<td>Elsmore 1968</td>
</tr>
<tr>
<td>750</td>
<td>$0.39 \pm 0.06$</td>
<td>Menon and Terzian 1965</td>
</tr>
<tr>
<td>1407</td>
<td>$1.3 \pm 0.13$</td>
<td>Elsmore 1968</td>
</tr>
<tr>
<td>1410</td>
<td>$1.48 \pm 0.05$</td>
<td>Menon and Terzian 1965</td>
</tr>
<tr>
<td>1420</td>
<td>$1.3 \pm 0.07$</td>
<td>Thompson et al. 1967</td>
</tr>
<tr>
<td>2695</td>
<td>$3.44 \pm 0.05$</td>
<td>Davies 1968</td>
</tr>
<tr>
<td>2840</td>
<td>$3.53 \pm 0.3$</td>
<td>Thompson et al. 1967</td>
</tr>
<tr>
<td>3000</td>
<td>$6.2 \pm 0.9$</td>
<td>Menon and Terzian 1965</td>
</tr>
<tr>
<td>4995</td>
<td>$6.5 \pm 0.6$</td>
<td>Present Result</td>
</tr>
<tr>
<td>4995</td>
<td>$6.50 \pm 0.14$</td>
<td>Davies 1968</td>
</tr>
<tr>
<td>5000</td>
<td>$5.96 \pm 0.35$</td>
<td>Hughes 1967</td>
</tr>
<tr>
<td>8000</td>
<td>$6.07 \pm 0.20$</td>
<td>Ehman 1967</td>
</tr>
<tr>
<td>15400</td>
<td>$6.0 \pm 1.0$</td>
<td>Kellerman and Pauliny-Toth</td>
</tr>
<tr>
<td>16200</td>
<td>$7.42 \pm 0.39$</td>
<td>Ehman 1967 (1967)</td>
</tr>
<tr>
<td>31400</td>
<td>$6.7 \pm 0.9$</td>
<td>Terzian 1969</td>
</tr>
</tbody>
</table>

**Table 11.** Flux Densities of NGC 7027
NGC 7027

Flux Density ($10^{26}$ Wm$^{-2}$Hz$^{-1}$)

300 1,000 3,000 10,000 30,000
Frequency (MHz)

FIG. 13. The radio spectrum of NGC 7027. The dotted line is the calculated spectrum of the model with a Gaussian density distribution (see text).
the brighter to the north are 7.5" apart in p.a. 135°. [A long exposure] shows an irregular oblong 18" x 11", with central details entirely 'burnt out'." Estimates of the size by other observers have tended to be rather smaller (see Perek and Kohoutek (1967) for summary), the most recent, by Perek (1963), being 13.7" x 10.6".

Unfortunately no optical isophotes have been published, nor an illustration indicating the relationship between the various parts of the nebula and the AGK2 position. The scale of the Sky Survey Atlas is too small to allow a useful comparison of radio and optical positions, especially since the image of the nebula is heavily overexposed on the prints.

It is very significant, however, that the position angles of the optical and radio sources are in agreement, since Elsmore's map of NGC 6720 (1968) has demonstrated that correspondence between optical and radio contours of planetary nebulae cannot be taken for granted, even when self absorption is absent. The agreement in the case of NGC 7027 strongly suggests that significant radio emission is coming from the fainter south eastern part of the nebula, rather than only from the fairly symmetric bright north-western condensation. Since they have comparable surface brightness, the north east filaments probably contribute
significantly to the radio emission as well, with the result that the centroid of the radio map will be some way to the east of the brightest part of the source.

The discrepancy between the radio position and that of AGK2 is now explicable if the optical position refers to, or is biased towards, the brightest part of the nebula. The fact that Perek and Kohoutek see fit to quote a position to ± 0.1" arc does indeed suggest that the measurement was made on a very small image, but this is probably a dangerous assumption to make without details of the astrometric methods used. Nevertheless Elsmore (1968) notes that for the planetary NGC 6853 the peak of the radio emission corresponds more closely to the centre of the nebula than to the optically brightest region, just as now seems to be the case with NGC 7027.

4. Physical Conditions in the Nebula.

The resolution of the present observations is sufficient to allow an estimate to be made of the electron temperature, based on surface brightness. For the reasons discussed in Chapter II and Appendix A, Gaussian widths are not the most useful parameters for determining electron temperatures, so a modelfitting procedure was used.

In the case of NGC 7027 the sharpness of the turnover
region in its radio spectrum puts a severe restriction on the possible models, as sources containing a wide variation of emission measure will produce a much more gradual change of slope than that in Figure 13. The dotted line in Figure 13 is the spectrum of a model comprising a spherically symmetric Gaussian density distribution. The agreement between it and the actual spectrum is very poor and cannot be significantly improved by altering the physical parameters of the model, such as electron temperature or half-width. A model comprising a sphere of gas of uniform density produces a spectrum very close to the experimental curve which indicates that the electron density in the nebula falls to zero quite rapidly and that the nebula does not incorporate extensive low density regions. In view of the observed filamentary structure of the nebula this is surprising, but without optical isophotes the precise radio spectrum of the source is difficult to predict.

A more extensive analysis of this problem is planned for the future as it seems likely that useful quantitative restrictions on the dispersion of the electron density distribution could be made from detailed knowledge of the spectrum of the source in its turnover region. Even a cursory examination of planetary nebula photographs shows an enormous range in the extent to which the emission is
from filaments and condensations, so that any information about the dispersion of the electron density will provide important data about the physical conditions in cases where optical information is scant or non-existent, such as cocoon stars or small planetaries.

A series of models comprising a uniform density sphere if ionised gas was then investigated. For each electron temperature the density and diameter were chosen to fit the radio spectrum and the model was convolved with the known elliptical beam of the One Mile Telescope, making the correct allowance for finite optical depth. The central brightness temperature of the resultant maps varied sharply with electron temperature, and the best agreement was obtained for a model of diameter 10" arc and $T_e = 15000^\circ K \pm 2000^\circ K$. A distance of 1.77 kpc was assumed for the nebula (O'Dell, 1962) which leads to a diameter of 0.083 pc and an electron density of $3 \times 10^4$ cm$^{-3}$.

The model is clearly a simplification in view of the known inclination of the major axis of the radio source, but when allowance is made for the foreshortening of the map in declination the agreement for all but the very lowest contours is remarkably good. Wilson's (1950) study of the splitting of spectral lines is indicative of some kind of shell structure in the source, although there is little
direct sign of this in Plate II. A model comprising a shell of thickness equal to half its width was tried: the best fit required a slightly higher electron temperature (16000°K) but it was not possible to distinguish between the two models on the basis of either the radio contours or the spectrum.

The electron temperature obtained here agrees very well with the optical determination by Aller (1954) of 15000°K and, within experimental error, with the 17000°K derived by Seaton (1960). The diameter is such as to correspond with the size of the two lower images in Plate II rather than just the bright condensations in the short exposure pictures. This is in agreement with the conclusions drawn in section 3. Aller (1954) found that the electron density varied from $10^4$ to $2 \times 10^5$ cm$^{-3}$, with an average of $7 \times 10^4$. The discrepancy between that value and the $3 \times 10^4$ of this paper is partly accounted for by the different averaging procedure and partly because no allowance for any filamentation has been made in the radio model.

From observations of the 5 GHz H109α recombination line Mezger et al. (1967b) inferred an temperature of 11000°K $\pm 3000°K$ which is significantly lower than both the optical and the present radio measurements. It has been shown by Mansfield (1969) that if the temperature in the nebula is
not uniform optical determinations will be biased towards the hot and radio line measurements towards the cool regions due to the different averaging procedures used. The present measurement is biased towards the outer regions of the source, as it involves knowledge of the optically thick part of the spectrum.

In the case of NGC 7027, however, there is no reason to expect large temperature variations and the evidence, such as there is, supports the view that the nebula is fundamentally isothermal. Firstly Aller and Csyzak (1968) remark on the absence of stratification in the optical spectrum, which is suggestive, but not proof, of constancy of temperature, and secondly the spectral index of the optically thick part of the radio spectrum is very close to $-2$. If the electron temperature had been falling away from the centre of the nebula, for example, a steeper spectrum would have resulted, since at progressively lower frequencies only radiation from the outer regions would be received. For a difference in average temperature of 4000$^\circ$K to arise, this effect would almost certainly be noticeable if the temperature was primarily a function of distance from the exciting star. It thus seems more likely that the low value for the radio recombination line temperature is due to departures from thermal equilibrium in the source.
as discussed by Dyson (1967) and others. Observations of other radio lines would be useful in establishing the degree of departure from equilibrium, as NGC 7027 is the most compact source to have had temperature measurements made based on both optical and radio line intensities.

5. Conclusions.

The data from the new observations described in this chapter are all consistent with the known physical conditions in the nebula, and the optically derived temperature has been confirmed by direct measurements of the radio brightness of the source. Although the identification of NGC 7027 as a planetary has never been seriously in doubt the similarity of its spectrum to that of a Seyfert Galaxy in both the optical and infra-red has been remarked on (e.g. Osterbrock et al. 1968). The fact that the radio size of the object is now seen to be very close to that explicable by free-free emission at 15000°K is evidence against the Seyfert Galaxy hypothesis, since the agreement in that case would be purely fortuitous. The radio size is comparable with the overall optical size, but there is a certain amount of evidence to suggest that the radio source has less of a filamentary character than the optical.

As Mezger et al. (1967b) pointed out, there is very
little to distinguish a planetary nebula from a compact HII region at radio wavelengths. The masses of condensations vary greatly (Chapter VI) and, in view of the high value derived for W 49, distinction on the grounds of electron temperature is dangerous, even though that of NGC 7027 is nearly twice that of DR 21.

What will be really interesting is a comparison of the optical properties of NGC 7027 with those of HII condensations, now that two components of W 58 are visible. The abundances of the elements are quite likely to vary, for example. It is not impossible that such a study will lead to the discovery that other supposed planetaries are in fact much younger objects, like NGC 6857.
VI. THE EVOLUTION OF COMPACT HII REGIONS

1. Condensations in other HII regions.

The compact HII regions in W 49, W 58 and W 75 have been discussed in Chapters II - IV. The existence of similar objects in W 3, NRAO 591 and M 8 has been proposed by Mezger et al. (1967b), Hughes and Butler (1969a) and Davidson and Harwit (1967) respectively. These have not yet been observed in Cambridge for various reasons: the condensation in W 3 is part of a very extended source which would require a very large amount of observing time at any frequency: NRAO 591 is at low declination and is a very recent discovery: M 8 is a southern nebula innaccessible to the One Mile Telescope. It is not certain that the condensation in M 8, the optical feature known as the 'hourglass', is of the same type as the radio sources, and it will not be considered here. The parameters of the condensations in W 3 and NRAO 591 are listed in Table 12. Hughes and Butler considered a Gaussian density distribution in their modelfitting procedure. Here a spherical uniform density distribution has been assumed for consistency with the other sources. The condensations in W 49 are not considered in this chapter since
2. The Density-Depth Ratio Relation.

<table>
<thead>
<tr>
<th>Diameter (pc)</th>
<th>Electron Density (cm⁻³)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>W 3</td>
<td>0.06</td>
<td>30 000</td>
</tr>
<tr>
<td>NRAO 591</td>
<td>0.27</td>
<td>17 000</td>
</tr>
</tbody>
</table>

Table 12. Data for other known condensations

Mezger et al. (1967a) showed that six or more stars would be necessary to produce the observed ionisation. Until the sizes and number of condensations in W 49 are known, discussion of their evolution is rather difficult.
Mezger et al. (1967a) showed that six or more stars would be necessary to produce the observed ionisation. Until the sizes and number of condensations in W 49 are known discussion of their evolution is rather difficult.

2. The Density-Diameter Relation.

Figure 14 shows the electron density of the known condensations as a function of their diameters for W 3, NRAO 591, DR 21 and the four components of W 58 (labelled A, B, C and NGC 6857). The open circle labelled A' shows the effect of distance uncertainties on the properties of the sources: A' is the position of A on the diagram if the distance to the source is only half that assumed in Chapter IV. Lines of constant mass and of constant excitation parameter are shown. The line AA' is a line of constant emission measure.

All the condensations can be excited by a single 0 type star, some properties of which are listed in Table 13. Most of the data are from Rubin (1968) but the masses are from Mezger and Palmer (1968), calculated by Rubin, and the main-sequence lifetimes have been estimated from the data collected by Gottlieb and Upson (1968). It can be seen that an 06 star or earlier is necessary to excite most of the condensations, although it is possible
FIG. 14. The electron density of the known HII condensations plotted against their diameter. A' is the position of component A of W 58 if its distance from the sun is only half that assumed in chapter IV.
<table>
<thead>
<tr>
<th></th>
<th>$T_*$</th>
<th>$S_*$</th>
<th>Mass</th>
<th>Main sequence Lifetime ($10^6$ y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($10^3$K)</td>
<td>(pc cm$^{-2}$)</td>
<td>(M$\odot$)</td>
<td></td>
</tr>
<tr>
<td>04</td>
<td>50</td>
<td>87</td>
<td>53</td>
<td>1.6</td>
</tr>
<tr>
<td>05</td>
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<td>5.6</td>
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<tr>
<td>08</td>
<td>36</td>
<td>30</td>
<td>19</td>
<td>6.5</td>
</tr>
<tr>
<td>09</td>
<td>35</td>
<td>26</td>
<td>18</td>
<td>7.1</td>
</tr>
<tr>
<td>09.5</td>
<td>33</td>
<td>18</td>
<td>16</td>
<td>8.6</td>
</tr>
<tr>
<td>B0</td>
<td>31</td>
<td>11</td>
<td>14</td>
<td>10.7</td>
</tr>
<tr>
<td>B0.5</td>
<td>29</td>
<td>6.8</td>
<td>13</td>
<td>13.0</td>
</tr>
</tbody>
</table>

**Table 13.** Physical parameters of early type stars.
that the excitation could be by a binary system of slightly later stars without greatly affecting the conclusions drawn in this chapter.

Davidson and Harwit (1967) have shown that the time for the formation of the initial Strömgren sphere around a cocoon star is only of the order of 20 years, so it is safe to assume that the visible HII condensations are in the expansion stage, their radii increasing due to the pressure in the hot ionised regions. When an HII region expands its density and its emission measure decrease. Depending on whether it is ionisation or density bound it will follow a line of constant excitation parameter (i.e. constant rate of ionisation and recombination) or constant mass. Figure 14 demonstrates that while the masses of ionised hydrogen in the known condensations vary by a factor of nearly 300 the excitation parameters vary by a factor of only 3. If, as seems most probable, B, C and NGC 6857 are later stages in the evolution of the more compact HII condensations it must be concluded that these objects are ionisation bound and that the total mass of the dense gas into which the ionisation front is expanding is several tens of solar masses.

This is a very important result as previous theoretical studies of the dynamics of HII condensations (Davidson and
Harwit 1967; Mathews 1968) have tended to assume that the mass of the cloud was much smaller, only a few $M_\odot$. This means that in Mathews' model, for example, which assumes a $3M_\odot$ circumstellar cloud, the HII region is density bound for most of its visible life. Its excitation parameter would thus be significantly lower than that of its exciting star whereas, in W 58, the excitation parameters observed are already those of the very brightest stars known. It would, however, be most interesting to obtain very high resolution HI line measurements of the vicinities of the most compact condensations to confirm that they too are surrounded by a total mass of several tens of solar masses.

3. Time scale of evolution.

In the previous section evidence was produced to show that the HII condensations are expanding into localised regions of fairly dense neutral gas. It is rather dangerous to discuss the evolution of these objects without making allowance for that of the stars with which they were formed, but, like Mathews (1968) and Davidson and Harwit (1967), it will be assumed here that in each case the central star attains its full luminosity in a time short compared with the expansion of the HII region. This assumption will be justified later.
The dynamics of the expansion of diffuse HII regions has been discussed extensively elsewhere, for example by Spitzer (1968b), and the basic theory may be applied to condensations without any difficulty. After the initial formation of the Strömgren sphere a shock wave forms which precedes the ionisation front and both compresses and accelerates the interstellar medium. The two fronts move at nearly the same velocity, which is given approximately by:

\[ \frac{dR}{dt} = C_{II} \left( \frac{\rho_{II}}{\rho_{I}} \right)^{\frac{1}{2}} \]  

(1)

where \( C_{II} \) is the velocity of sound in the ionised medium and \( \rho_{II} \) and \( \rho_{I} \) are the densities on the ionised and neutral sides of the pair of fronts. \( R \) is related to \( \rho_{II} \) by the fact that \( R_{II}^{2/3} \) is effectively the excitation parameter of the exciting star and is therefore constant. \( \rho_{I} \), the density of the unshocked neutral gas, depends on the initial conditions in the circumstellar cloud and is basically an unknown quantity. If, however, it is assumed that the neutral cloud is of uniform density \( \rho_{o} \), significantly higher that \( \rho_{II} \), equation 1 can be integrated to give:

\[ R = \frac{7}{4} C_{II} \left( \frac{\rho_{II}(t)}{\rho_{o}} \right)^{\frac{1}{2}} t \]  

(2)
At an electron temperature of $10^4 \text{K}$, $c_{\text{II}} = 11 \text{ km/sec}$, and the time for the diameter of the HII region to reach 1 pc is given by:

$$t (10^6 \text{ years}) = \frac{1}{40} \left( \frac{\rho_0}{\rho_{\text{II}}} \right)^{1/2}$$  \hspace{1cm} (3)

The initial density $\rho_0$ is unknown but if it is of the same order as that of the densest condensations it means that the largest components in W 58 are of the order of $10^5$ years old. This time is significantly longer than the "turn on" time of an O star, which is less than 1000 years (Davidson and Harwit 1967) and is also significantly shorter than the main sequence lifetime of any likely exciting stars (Table 13), so that the neglect of stellar evolution is justified as a first approximation. However the estimate of age is very uncertain indeed and represents neither an upper nor lower limit. If the initial density $\rho_0$ were larger the time scale would be longer but this is difficult to test since even if condensations denser that W 3 exist they would be difficult to detect owing to their consequent very high turnover frequencies. Conversely if the HI density decreases away from the central star the ionisation front will move more rapidly and the time scale for expansion would be reduced. Nevertheless the present estimate of the expansion rate is in accord with Reddish's (1968)
estimate for the lifetime of circumstellar dust clouds as between $10^5$ and $2 \times 10^6$ years. Reddish also calculated the gravitational free-fall collapse time of such a cloud to be $8 \times 10^5$ years, but this, again, depends on the value taken for its density.

In the next section reasons are given for believing that the total masses of the circumstellar clouds are not much more than the masses of the larger components in Figure 14. If this is so, the dynamic evolution at this stage is rather more complicated since the effects of the ionisation fronts between the outside of the HI cloud and the surrounding diffuse HII region should be considered. Optical studies of the morphology of NGC 6857 may well shed light on this problem in the near future.

Finally the only direct evidence concerning the time scales of evolution is the measurement of 25 km/sec for the RMS turbulent velocity in DR 21 (Mezger et al. 1967b). This is compatible with the expected velocities associated with shock fronts (Equation 1) but, in view of the strong possibility of rotation (section 4) it is not wise to interpret the line width as an indication of any particular expansion velocity in the condensation. Neither is it possible to draw any conclusions from the line profile, although higher frequency resolution might yield very useful data.
4. Relevance to star formation.

The condensations observed in the present work are all associated with the most luminous sorts of stars known, having excitation parameters characteristic of the very earliest O stars. There is certainly a very strong selection effect operating here but the absence of any smaller condensations in the vicinity of either DR 21 or NGC 6857 may well be due to the fact that in a fragmenting cloud the most massive stars form first, after a rapid proto-star evolution, and the cocoons around them dissipate by the time the slightly later stars form.

The work of Reddish (1967 and 1968) is very relevant here. From a study of the relationship between colour excess and luminosity among OB stars he found that the stars in young clusters tend to be surrounded by circumstellar clouds with an average mass of $27 \, \text{M}_\odot$ and a diameter $< 0.8 \, \text{pc}$. Bearing in mind the uncertainties in the distance of W 58 this is remarkably close to the values of the masses of the larger components of that source, and suggests that the ionisation bound stage of these regions is nearly over.

Reddish also cites the theoretical work of Ireland (1967) who has shown that for the most efficient transfer of angular momentum between a collapsing star and its associated cloud the masses of the star and cloud should be
nearly equal, and that over a wide range of conditions the most probable situation is for 40 - 60% of the total mass of the system to be contained in the cloud. It can be seen from the data in Table 13 that this is just the case with the larger condensations in W 58, and, again, suggests that their circumstellar clouds are now almost all ionised, whereas those of the more compact condensations are still only partially so.

If the sizes and the masses of the circumstellar clouds are governed by the requirements of angular momentum transfer the dynamics of the shock and ionisation front propagation is going to be very complicated and will require much careful thought in the future. The assumption of spherical symmetry of the gas cloud obviously becomes very dangerous under these circumstances and, as seen in Figure 7, DR 21 shows definite signs of being flattened. Rotational forces may also explain why the optical obscuration of similar sized objects such as DR 21 and W 58A, and NGC 6857 and W 58 C varies so much, since the amount of dust in the line of sight would depend very much on the direction of the rotation axis.

The state of the visible condensations is thus closely related to the conditions surrounding the formation of its star, so it now seems that radio astronomy can study early
stellar evolution and provide data which is not available in any other way. This is because, except at the point where light from the star itself is transmitted through the cocoon, the dusty circumstellar clouds tend to be invisible optically whereas at radio wavelengths the HII region is clearly visible and its structure, and possibly the dynamics of the cloud, can be inferred. A search for and study of more HII condensations would therefore be of great importance to the study of this very crucial interaction between the stars and the interstellar medium.

Studies of condensations in HII regions may be relevant to theories of star formation in another way. If several condensations are visible at once their relative positions provide useful data about the distribution and movement of the interstellar gas out of which they condensed. W 58 is the only group of condensations in which this is possible as yet but, even so, it is a rather interesting example as the condensations are extended along a line approximately parallel to the galactic plane, as shown by the arrow in Figure 10. The galactic latitude here is about +1.6° which suggests that the location of the new stars is influenced by galactic motions. Also, in view of the remarkably straight line joining components A, B and C it is interesting to note the proposal put forward by Fessenkov and Razhkovsky
in 1952 that in many cases groups of new stars are formed in a line, thereby producing a star chain, but, according to Spitzer (1968a) this suggestion has not been widely accepted.

HII condensations, therefore, may well provide important new data on these questions since massive stars may thereby often become visible in the radio region some time before they are detectable by optical telescopes. It is also possible that measurement of the sizes and densities of HII condensations may help to identify stars that have formed simultaneously, but this will have to wait until a lot more data on groups of stars are available.

5. Future Observations and Present Conclusions.

There is no doubt that the most important requirement for the continued study of HII condensations is the discovery of more sources. All the known condensations are associated with bright diffuse HII regions and most with OH sources, but it is impractical to use the One Mile Telescope to survey other HII regions looking for fine structure since most of them are large and in many cases up to three months observing time would be required to produce a map free of grating rings at any frequency. In addition many of the most interesting Galactic HII regions are in the Southern sky, or at
very low declination, inaccessible to the One Mile Telescope. The location of the Orion Nebula at $\delta = -4^\circ$ in particular is a sad blow for Cambridge radioastronomy, though it may be possible to make a one dimensional survey of the source in the future.

It is worth using the One Mile Telescope only on suspected sources that are unlikely to be too affected by interference from grating rings. The next step therefore is to look for likely candidates among known celestial bodies. Firstly it is important to establish more firmly the identity of HII condensations and circumstellar dust globules, and a study of some of Reddish's stars is clearly overdue. Next, although the connection between HII condensations and infra red stars is not yet clear, it is hoped that a comparison of the long-awaited Caltech infra red survey with radio source catalogues will lead to some possible candidates in known regions of star formation. Also, in view of the fate of NGC 6857, a careful study of planetary nebula data might be valuable although according to Thomasson (1969) NGC 6857 is the only source in the recent unpublished Jodrell Bank work on planetaries which was found to have a suspicious spectrum.

One of the difficulties about studying HII condensations is that the selection effects in their discovery are rather
uncertain and that they may not always be associated with OH emission or diffuse HII nebulosity. It will therefore be worth studying high frequency radio surveys of the galactic plane for objects with high cut off frequencies or flat spectra. Finally, of course, it is very much hoped that evidence for new condensations will be found by other radioastronomers using lower resolution instruments over a wide range of frequencies.

In the meantime 5 GHz observations of W 49A and, if possible, W 58 are planned. The W 49A condensations have not been resolved and their existence has only been inferred from spectral evidence. As mentioned before W 49A is especially interesting because of the degree of molecular activity there, as well as the fact that this is only the second opportunity to study a complete group of HII condensations.

Several interesting theoretical problems which require careful thought have been mentioned in the text, such as those connected with the radiospectra of condensations, their anomalous electron temperatures, the effect of rotation on an expanding HII region and the physical conditions necessary for the production of the intense OH and H$_2$O emission which may or may not be from the condensations themselves.
Thus although the physics of low density diffuse thermal nebulae is moderately well understood there is a great deal of theoretical and experimental work to be done on HII condensations. Their significance to stellar evolution and to the interstellar medium has already been discussed, but these objects are also most rewarding to study because of the variety of data available, potentially at least. Although primarily continuum radio sources with characteristic spectra, they produce radio recombination lines, are usually associated with molecular masers, are sometimes visible as emission nebulae and are sometimes close to infra red sources. Their effect is seen in the reddening of some early stars and they have to be studied in the context of the surrounding HI, HII and dust clouds. Since the condensations are excited by very hot stars there is also the exciting possibility that the ultra-violet data currently being accumulated by the Orbiting Astronomical Observatory will add to our knowledge of the sources.

Because of their inherent obscuration, however, radio-astronomy is likely to remain the most powerful tool for the study of HII condensations. In particular this dissertation has attempted to show the value of very high resolution radio astronomy in this field. Although radio interferometry can demonstrate the presence of fine structure, aperture
synthesis on the scale of the One Mile Telescope is necessary for measuring the sizes of the condensations and locating them accurately enough for comparison with infra-red, optical and OH emission sources to be made. Beamwidths of a few seconds of arc are vital if groups of condensations are to be resolved and their densities and hence their mode of evolution are to be determined.

At the present the Cambridge One Mile Telescope is the only instrument that can provide this data.
VII. THE PHASE STABILITY OF COAXIAL CABLES IN RADIOASTRONOMY.

1. Introduction.

As their titles imply this chapter and the next are not concerned very directly with Galactic ionised hydrogen condensations. It has already been stressed, however, that the main value of the observations described in Chapters II - IV is their high resolution, which in every case is several times greater than that previously achieved: the conclusions reached in Chapter VI depend crucially on being able to resolve the components of a group of condensations. A single dish can rarely achieve half power beamwidths narrower than 2' arc, so an aperture synthesis instrument such as the supersynthesis system developed at Cambridge is absolutely vital. The operation of an aperture synthesis telescope depends, among other things, on being able to measure accurately the relative phase between the signals received at small aerials spaced at distances apart up to the diameter of the synthesised aperture, which is one mile in the present telescope. For the phase to be accurately measurable the behaviour of the radio frequency coaxial cable links between the aerials must be well understood, in particular the variation of their electrical
lengths with temperature. It is with this problem that these two final chapters are concerned.

The One Mile Telescope works very well as it is, and was designed and built some time before the author had anything to do with radioastronomy. The present study is therefore mainly concerned with the next Cambridge instrument, the Five Kilometre Telescope, a larger and more elaborate system now under construction, working on the same principles as the One Mile Telescope. The variation in the cable lengths here will be much more serious as the lengths are longer and the operating frequency is higher so, as part of the design study for the new instrument, it was decided to reanalyse some of the problems associated with cables in radio telescopes in more detail than before. This has very largely been the work of the author.

In section 2 the necessary stability criteria for aperture synthesis are discussed, and in sections 3 and 4 the thermal behaviour of the cables used is studied both theoretically and experimentally. The results of the previous sections are then combined in section 5 which discusses the depth at which it is necessary to bury cables underground to minimise phase fluctuations and in section 6 where problems connected with cables above the ground on the aerial superstructure are discussed.
Chapter VIII is concerned firstly with the apparatus with which the experimental work in this chapter was carried out and secondly with the design of a new instrument capable of measuring the electrical lengths of the longest cables which are to be installed in the 5 Kilometre Telescope.

2. The Stability Required.

The One Mile Telescope was described briefly in Chapter I and in more detail in Elsmore et al. (1966). The instrument comprises one movable and two fixed 18 m paraboloid dishes and has been used for observations at wavelengths from 6 cm to 75 cm. The receivers are all housed in a control room close to the central aerial, so that the RF links from the two distant dishes are about 800 m long. The Five Kilometre Telescope will have eight dishes and is designed to operate at frequencies up to 10 GHz at a maximum spacing of 5000 m. In both instruments conversion of the incoming signal to an I.F. of 45 MHz is performed at the focus of each dish, the local oscillator signal being generated in the central control room and sent to each aerial via another coaxial cable. At the higher frequencies used (1.4 - 5.0 GHz) the local oscillator frequency transmitted is a subharmonic of that used in the mixing stage. Although this necessitates constructing phase
stable multipliers in each aerial the attenuation in the
cables is reduced to manageable proportions if frequencies
below 400 MHz can be used. However it can be shown that
whatever frequency is actually transmitted along the local
oscillator cables, a change $\delta x$ in their length produces
a phase change of $\frac{2\pi\delta x}{\lambda_s}$ where $\lambda_s$ is the wavelength of the radio
signal received. Thus when the telescope is operating at
10 GHz ($\lambda = 3$ cm) a variation of length of 1 cm in the local
oscillator cables produces a phase change of 120°, although
a similar disturbance in the I.F. cables ($\lambda = 670$ cm) repres-
ents a phase change of only 0.5°. The local oscillator
cables thus need to be compensated against length variation
much more than do the I.F. cables. This is done by
sending the L.O. signal to each dish, including the centre
one, through the same length of cable, buried at the same
depth. In this way, so long as no great temperature
difference is set up between the different parts of the
system the phase fluctuations will be much reduced.

Absolute knowledge of the path length from the focus
of each dish to the receiver is, fortunately, unnecessary
in aperture synthesis, since the collimation errors, which
represent the differences in these path lengths for each
pair of aerials, is determined accurately astronomically,
as described by Elsmore and Mackay (1969). What is necessary,
however, is that the collimation error should change as little as possible between such calibrations, and so in section 5 an investigation is made to determine at what depth the L.O. cables should be buried in the Five Kilometre Telescope for the collimation error to be constant to within 10° of phase at \( \lambda = 3 \text{ cm} \).

A different problem arises for the I.F cables since, although they are less sensitive to temperature changes than the L.O. cables, it is impossible to arrange that all the I.F. signals pass through the same length of buried cable, as it is for the local oscillator. This is because the I.F. path length must be chosen to compensate for the path difference between the aerials and the radio source (Elsmore et al 1966). For this reason the absolute lengths of the I.F. cables and path compensating cables must be known and it can be shown (Elsmore and Mackay 1969) that a change in the length of the path compensator cables is equivalent to a displacement of the radio source under observation. The displacement \( \Theta \) in the declination \( \delta \) is given by:

\[
\Theta = \frac{\alpha \phi}{0.81 \times 2\pi \cos \delta}
\]

where \( \alpha \) is the half power beam width and \( \phi \) the phase error in the path compensator at I.F. It will be shown in section 5 that this error is usually negligible.

Almost all the long R.F. links in the Cambridge radiotelescopes use BICC HM4 cable, although other short pieces of flexible solid polythene filled cable are used for linking purposes. HM4 is an air spaced cable, with a copper inner, an aluminium outer and a helical polythene spacer. It has a nominal impedance of 75 ohms and an overall diameter of about 2 cm. Its attenuation at 70 MHz is about 0.4 dB per 100'.

The effect of an increase of temperature is, of course, to cause expansion of the component parts of the cable. The electrical length may then change due to four effects;

i) The increase in physical length of the cable. This will be some weighted mean of the coefficients of expansion of copper and aluminium, but will be affected by the adhesion of the soil if the cable is buried.

ii) Decrease in $\varepsilon_p$, the dielectric constant of polythene, according to the Clausius–Mossotti equation.

iii) Alteration in the relative proportions of volume occupied by air and polythene.

iv) Alteration of the density of the air due to the change in volume inside the cable.

The electrical length ($L$) is given by:

$$L = L' \sqrt{\frac{(1-f)\varepsilon_a + f\varepsilon_p}{\varepsilon_p}}$$
where $L'$ is the physical length, $\varepsilon_a$ is the dielectric constant of air and $f$ is the fraction of volume between the conductors filled with polythene.

Let $K = \varepsilon_a - 1$

$$\eta = \frac{1}{L ' \frac{dL'}{dT}} + \frac{1}{2c} \left[ \frac{d(f(\varepsilon_p - 1) + \frac{d(K(1-f))}{dT})}{dT} \right]$$  

\[ (2) \]

where $\eta$ is the coefficient of expansion of electrical length. The fraction of volume occupied by polythene is given by:

$$f = \frac{m}{\rho V}$$

where $m$ is the total mass of polythene, $\rho(T)$ is the density of polythene and $V(T)$ is the volume between conductors. The Clausius-Mossotti equation states that

$$\frac{\varepsilon_p - 1}{\varepsilon_p + 2} = \frac{4\pi N \alpha \rho}{3 M} = A^p$$

where $A$ is some constant.

$$f(\varepsilon_p - 1) = \frac{3 \pi m}{V(1 - A^p)}$$

and

$$\frac{d(f(\varepsilon_p - 1))}{dT} = f(\varepsilon_p - 1) \left[ -\beta(\varepsilon_p - 1) - \frac{1}{V} \frac{dV}{dT} \right]$$  

\[ (3) \]

where $\beta = - \frac{1}{\rho} \frac{d\rho}{dT}$, i.e. the volume coefficient of expansion of polythene under the appropriate conditions.

The dielectric constant of air is given by $\varepsilon_a = 1 + \frac{aP}{T}$

$$\frac{d(K(1-f))}{dT} = \frac{d}{dT} \left[ \frac{aP(1-f)}{T} \right]$$

$$= -K(1-f) \frac{1}{V} \frac{dV}{dT}$$  

\[ (4) \]

$P$ is the gas pressure and $a$ is a constant.
At normal temperatures and pressures $K \approx 5 \times 10^{-4}$ so that this term can be neglected compared to the coefficient of $\frac{1}{V} \frac{dV}{dT}$ in equation 3. For HM 4 cable $f = 0.10$ and $\varepsilon_p = 2.26$, so that from equations 2 and 3:

$$\eta = \alpha - 0.025\beta - 0.060\frac{1}{V} \frac{dV}{dT} \quad (5)$$

where $\alpha$ is the coefficient of linear expansion of the cable. The relevant mechanical data are:

Coeff. of lin expansion of Aluminium = 23 ppm/°C
" " " Copper = 17 ppm/°C
" " " Polythene = 180 ppm/°C

Poisson's Ratio of Aluminium ($\sigma_a$) = 0.345
" " " Polythene ($\sigma_p$) = 0.2

The polythene is severely restricted from expanding perpendicular to the axis of the cable so that

$$\beta \approx 180 \frac{1 + \sigma_p}{1 + \sigma_p} \approx 250 \text{ ppm/°C}$$

We must consider separately the cases of whether or not the cable is being restrained from expanding physically.

If it is not restrained $\alpha = 23 \text{ ppm/°C}$, $\frac{1}{V} \frac{dV}{dT} = 69 \text{ ppm/°C}$ since the aluminium has a much greater cross section than the copper and will dominate in the expansion,

$$\eta = 23 - 6 - 4 = 13 \text{ ppm/°C}$$

If it is restrained $\alpha = 0$, $\frac{1}{V} \frac{dV}{dT} = 23(2(1+\sigma_a)) = 62 \text{ ppm/°C}$ and

$$\eta = -6 - 4 = -10 \text{ ppm/°C}$$
If the cable is bolted to the steel framework of the telescope and forced to expand with the steel \( \alpha = 11 \text{ ppm/}^\circ\text{C} \).

and \( \eta = 11 - 6 - 4 = 1 \text{ ppm/}^\circ\text{C} \).

The main source of error in the calculation is the difficulty of predicting the behaviour of polythene, since it is very difficult to know, yet alone to know how to deal with, the stresses and anisotropies introduced during manufacture. An error of 30\% is not unreasonable from this cause. This represents \( \pm 2 \text{ ppm/}^\circ\text{C} \) so that for HM4;

If the cable adheres to the clay \( \eta = -10 \pm 2 \text{ ppm/}^\circ\text{C} \)

If the cable is free to expand \( \eta = +13 \pm 2 \text{ ppm/}^\circ\text{C} \)

If the cable is bolted to steel \( \eta = +1 \pm 2 \text{ ppm/}^\circ\text{C} \)

The theory in this section is also applicable to polythene filled cables by putting \( f = 1 \) and \( \frac{1}{V} \frac{dV}{dT} = \beta \) in equation 3. This is because the braided outer, being flexible, will be forced to expand outwards with the polythene. These figure give, using equation 3,

\[ \eta = \alpha - 0.40\beta \] 

(6)

In this case \( \alpha \), the coefficient of expansion along the length of the cable, will be somewhere between that of polythene and of copper. If the polythene is dominant \( \eta = -40 \text{ ppm/}^\circ\text{C} \); if the copper is dominant \( \eta = -170 \text{ ppm/}^\circ\text{C} \). The precise value will depend on the way the conductor is
braided and will probably be intermediate between the two values in most cases. The electrical length of a solid cored polythene cable always contracts on heating, but that of a polythene spaced, air cored cable may expand or contract, depending on its construction.


The electrical length of half a mile of HM4 cable buried at a depth of 2.5 feet in blue clay was monitored for about six weeks in the spring of 1968. One of the main purposes of this experiment was to determine the extent to which the cable adhered to the clay. The apparatus used is described in the first section of Chapter VIII. The earth temperature in the vicinity of the cable was recorded every few days using a mercury thermometer. Over the six weeks the electrical length decreased by 3.5 cm while the earth temperature rose by 5°C. This means that experimentally

\[ \eta = -8 \pm 2 \text{ ppm/°C.} \]

for buried HM4.

The error is mainly due to the unreliability of the temperature measurements. Comparison with the theory in section 3 shows that by and large the cable is restrained by the clay, though a small amount of physical expansion is possible within the limits of the experiment. The main effect of temperature, though, is the change in the dielectric
constant of polythene.

In addition tests were made on lengths of solid polythene flexible cables. Most of the cables in use as short links are made of 75 ohm BICC UR 57 cable which has a solid copper inner conductor, braided copper outer and a PVC sleeve. Recently use has been made of UR 59 cable, which is identical except that it has a polythene sleeve. The effect of this is to bind the copper braiding more firmly, making the cable as a whole much stiffer than the UR 57. Experiments were made to compare these cables by measuring the changes in their electrical lengths due to various stresses. These showed that the length stability of UR 59 was better by a factor of about 3 when the cables were subjected to torsion and about 10 when being coiled. Under many applications these factors are further improved owing to the fact that the increased stiffness of UR 59 will reduce the strains involved to a minimum. Unfortunately the thermal properties of UR 59 are inferior to those of UR 57, the coefficients of expansion of electrical length being:

\[ \eta = -75 \pm 20 \text{ ppm/°C} \quad \text{for UR 57} \]

\[ \eta = -110 \pm 20 \text{ ppm/°C} \quad \text{for UR 59} \]

These measurements were made on loosely coiled cables with the apparatus described in Chapter VIII and between 10 and 30 metres of cable. It can be seen by comparing
these results with the theory in the previous section that the effective coefficient of linear expansion for both the cables is intermediate between that of copper and of polythene, but that in the more tightly bound UR 59 the physical expansion of the cable is restricted by the copper, whereas in the UR 57 the copper braid is freer to accommodate the larger expansion of the polythene.

5. Phase Variations in Underground Cables.

This section contains the results of an analysis made to determine the depth at which the local oscillator cables in the Five Kilometre Telescope should be buried. As described in section 2 the local oscillator cables are self compensating to some extent inasmuch as the signal to each aerial passes through the same length of cable buried under what are hoped to be identical conditions. It is with the likely deviations from this ideal situation that this section is concerned, due to variations of the thermal diffusivity of the soil beneath which the cables are buried.

The one-dimensional diffusion equation (7) governs the transport of heat through the soil, but it is only an approximation since the thermal diffusivity $k$ tends to vary with the depth $z$ to some extent, especially in the
top 10 cm or so (van Wijk 1963). Without a great deal of data on the soil at all points along the cable runs, however, refining of the mathematics is not justified.

\[
\frac{\partial^2 T}{\partial z^2} + \frac{1}{k} \frac{\partial T}{\partial t} = 0 \quad (7)
\]

This has solutions of the type:

\[
T = T_f \exp \left[-\left(\frac{\pi f}{k}\right)^2\right] \sin \left[\left(\frac{\pi f}{k}\right)^\frac{1}{2} z - 2\pi ft\right] \quad (8)
\]

where \( T_f \) is the amplitude of the Fourier component of the surface temperature with frequency \( f \), expressed in cycles per year.

In addition to the main Fourier components at \( f = 1 \) and \( f = 365 \) (corresponding to annual and diurnal variations respectively) intermediate frequencies, corresponding to alternate spells of warm and cold weather become relevant at the depths involved in this problem. To obtain an estimate of their importance a Fourier analysis was performed using Meteorological Office data for the maximum daily temperature at Kew, London, for the years 1964-6. The results, which can be assumed typical for southern England, have considerable spread but show that \( T_1 \), the amplitude of

---

FIG. 15. Fourier spectrum of the daily temperature 1964 - 1966. The horizontal scale is linear in frequency but is more conveniently expressed in terms of the period of the variation expressed in days, weeks or months.
the seasonal variation is 5 ± 1°C while 2 is fairly constant at around 0.7 ± 0.2°C over the range of 12 days, and the temperature drops off rapidly before rising again to about 5°C as 2 = 365, the diurnal component. (Figure 15).

This means that temperature variations due to the weather are not likely to occur with shorter periods. Because the attenuation increases with frequency the 12-day Fourier component is the most troublesome of all. After the annual and diurnal ones.

Since the site of the telescopes is flat and open, it can be assumed that the ground temperature is uniform at any time, so that it is with geographical variation that we are concerned, and an estimate of the range of likely to be encountered must be made before choosing the most suitable value of 1. The telescopes at Cambridge are built on clay and, although the ground is almost always fairly solid, considerable difference might be expected to occur between earth that has been excavated for levelling purposes and undisturbed clay. Table 15 gives values for the thermal diffusivity of clay and sandy soils. In view of these figures, and less accurate measurements.
the seasonal variation is $8 \pm 1^\circ C$ while $T_f$ is fairly constant at around $0.7 \pm 0.2^\circ C$ over the range $f = 3$ to $f = 30$ (periods of between 4 months and 12 days), and thereafter drops off rapidly before rising sharply to about $5^\circ C$ at $f = 365$, the diurnal component. (Figure 15). This means that temperature variations due to the weather may have any period longer than 12 days but are progressively less likely to occur with shorter periods. Because the attenuation increases with frequency the 12-day Fourier component is the most troublesome one after the annual and diurnal ones.

Since the site of the telescope is flat and open it can be assumed that the ground temperature is uniform at any time, so that it is with geographical variation of $k$ that we are concerned, and an estimate of the range of $k$ likely to be encountered must be made before choosing the most suitable value of $z$. The telescopes at Cambridge are built on clay and, although the ground is almost always fairly moist, considerable differences might be expected to occur between earth that has been excavated for levelling purposes and undisturbed soil. Table 15 gives values for the thermal diffusivity of clay and sandy soils. The two samples of clay are from different locations.

In view of these figures and less accurate estimates
<table>
<thead>
<tr>
<th>Volume Water Content</th>
<th>Thermal Diffusivity 10^3 cm^2 sec^-1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sand</td>
</tr>
<tr>
<td>0.43</td>
<td>8.7</td>
</tr>
<tr>
<td>0.40</td>
<td>-</td>
</tr>
<tr>
<td>0.30</td>
<td>10.3</td>
</tr>
<tr>
<td>0.20</td>
<td>11.5</td>
</tr>
<tr>
<td>0.15</td>
<td>12.1</td>
</tr>
<tr>
<td>0.10</td>
<td>12.5</td>
</tr>
<tr>
<td>0.05</td>
<td>10.6</td>
</tr>
<tr>
<td>0.00</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Table 14. Thermal diffusivity of clay and sand. The data are from van Wijk (1963) and van Wijk and de Vries (1963)

...
made at Cambridge (Baldwin and Jennings 1968) one is probably being fairly realistic in expecting $k$ to vary over the range $5 - 8 \times 10^{-3} \text{ cm}^2\text{sec}^{-1}$. Although one would be most unlucky to have a pair of cables buried such that one was entirely under earth of diffusivity $8 \times 10^{-3} \text{ cm}^2\text{sec}^{-1}$ and the other entirely under earth of diffusivity $5 \times 10^{-3} \text{ cm}^2\text{sec}^{-1}$ except under freak transitory drainage conditions, this possibility will be considered as a 'worst case' and the resultant phase changes between a pair of 2.5 km long cables at a wavelength of 3 cm will be calculated. The results are summarised in Table 15 and discussed in more detail below.

a) Diurnal Component. When the attenuation is large, as it is in this case, the sinusoidal term in equation 8 depends very critically on $k$ so that there will be no useful compensation by using a pair of cables. Table 15, therefore shows the possible diurnal phase changes when the diffusivity of the soil is at its highest ($k = 8 \times 10^{-3} \text{ cm}^2\text{sec}^{-1}$) in this case.

b) Annual Component. Although the attenuation increases the compensation becomes slightly less effective at lower depths. Table 15 shows the maximum phase change arising from this component in one day. This is largest in spring and autumn of course.
Table 15. Maximum possible phase variations per day between a pair of 2.5 km cables at $\lambda = 3$ cm due to diurnal, annual and 12-day components of the ground temperature variation.

<table>
<thead>
<tr>
<th></th>
<th>Depth (feet)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.0</td>
<td>3.5</td>
<td>4.0</td>
</tr>
<tr>
<td>Diurnal Component $^\circ$/day</td>
<td>6.4$^\circ$</td>
<td>2.2$^\circ$</td>
<td>0.9$^\circ$</td>
</tr>
<tr>
<td>Annual Component $^\circ$/day</td>
<td>2.5$^\circ$</td>
<td>2.7$^\circ$</td>
<td>2.9$^\circ$</td>
</tr>
<tr>
<td>Weather Component $^\circ$/day</td>
<td>9.1$^\circ$</td>
<td>7.3$^\circ$</td>
<td>6.1$^\circ$</td>
</tr>
</tbody>
</table>

The weather component is more likely to be important than the annual variation unless the cables are buried at a depth of more than four feet. This implies that during the operation of the telescope, the frequency of calibration observations should be related to the prevailing weather conditions, becoming less frequent during long spells of even temperature.

Because of the long I.P. wavelength, short-term temperature changes are not important if, at a depth of only two feet, the diurnal phase variation is less than one degree.
c) Weather Component. The phase variations are partially compensated for in the 12-day periodic oscillations, and the maximum phase changes arising per day are shown in Table 15. The total range of the fluctuation (pk-pk) is equal to four times the change per day in this case.

At a depth of 3' day-night variations of 6° phase would still be likely, but at 3.5' they are reduced to negligible proportions. The longer period variations are larger than this but are less of a problem, in that Table 15 represents 'worst cases' as discussed above. Under most circumstances the phase variations are unlikely to exceed a quarter of those tabulated. Table 15 also shows that the weather variation is more likely to be important than the annual variation unless the cables are buried at a depth of more than four feet. This implies that during the operation of the telescope the frequency of calibration observations should be related to the prevailing weather conditions, becoming less frequent during long spells of even temperature.

Because of the long I.F. wavelength short term temperature changes are not important at IF; at a depth of only two feet the diurnal phase variation is less than one degree.
of phase over 5 km. The seasonal variation is not much attenuated at depths of a few feet, however, which means that, over six months, a phase variation of $40^\circ$ will arise in a 5 km cable. As discussed in section 2 this phase change gives an error in declination of radio sources. At low declinations this is about 0.15 times the beamwidth, which is negligible except for the most precise measurements on small sources, when it might be worth making a small correction. This would involve monitoring the temperature in a representative cable trench.

6. Phase Variations in Cables on the Aerial Superstructure.

In the One Mile Telescope the L.O. and I.F. signals pass from ground level to the focus of each dish through Siemens flexible air spaced cable. The thermal properties of the cable are not dissimilar to those of HM4 but, being loosely fastened to the structure of the aerial, it can expand physically, so having a positive value for $\gamma$.

All the aerials being structurally identical, most of the length variations in these cables are compensated for automatically. However an experiment was performed to study the rapid fluctuations in length which arise as clouds intermittently shade the aerials on a sunny day. The length of these cables, approximately 40 m, was monitored
for a week in July 1968. There was typically a diurnal variation of 1 cm with fairly frequent jumps of 1 - 2 mm in 10 minutes as clouds passed overhead during the day. One occasion the length changed by 3 mm in 20 minutes but usually variations of this magnitude took more than an hour. One can conclude, therefore, that the time constant for the cable to be heated by the sun is around 5 - 10 minutes. With aerials spaced up to 5 km apart one must be prepared for variations like this whenever the sky is partially clouded in the summer except on very windy days. There are, however, several ways of improving the situation; i) The cables could be insulated with a reflective covering. In this way most of the fluctuations faster than an hour could be eliminated.

ii) As shown in section 3 the temperature coefficient of the cable could be reduced by using HM4 cable bolted onto the steel structure of the telescope. Also flexible cables are now available commercially in which the value of \( f \) in equation 3 has been chosen to make \( \gamma \) very close to zero.

iii) In the Five Kilometre Telescope the lengths of cable involved will be considerably shorter; since a Cass-eigrain feed system is to be used, which is nearer the ground. In addition the dishes are rather smaller
than those on the One Mile Telescope.

VIII. THE DESIGN OF CABLE MEASURING EQUIPMENT.

Thus the cabling on the superstructure of the aerials does not present a major obstacle to the development of large radio telescopes.

Measurements of the length of a cable are usually based on observations with a slotted line of the standing wave pattern set up in the cable by an H.F. oscillator. When there is considerable attenuation in the cable the VSWR becomes small and accurate estimations of the positions of the nodes become difficult, especially in the presence of small discontinuities close to the slotted line. This difficulty can be overcome by utilizing the reflection from the far end of the cable and employing a modulation sensitive detector which ignores all other reflections (Gawrap and Tung 1964). This principle is employed in the method developed by Khapire (1967) which was used for all the measurements described in Chapter VII.

The circuit is shown in Figure 16. The termination in this case is a semiconductor diode which is switched off and on at a frequency of 275 Hz by a signal from a square wave oscillator. The transmitted signal is of the form \( V_0 \cos(\omega t - k x) \) and the reflected signal will then be equal to \( AV_0 \cos(\omega t - k(2L - x) - \phi) \) where \( \lambda \) is the attenuation and \( \phi \) is either 0 or \( \pi \) depending on the state of the diode.
VIII. THE DESIGN OF CABLE MEASURING EQUIPMENT.

1. The Use of a Phase-Switched Receiver.

Accurate measurements of the length of a cable are usually based on observations with a slotted line of the standing wave pattern set up in the cable by an R.F. oscillator. When there is considerable attenuation in the cable the VSWR becomes small and accurate estimations of the positions of the nodes become difficult, especially in the presence of small discontinuities close to the slotted line. This difficulty can be overcome by modulating the reflection from the far end of the cable and employing a modulation sensitive detector which ignores all other reflections (Swarup and Yang 1961). This principle is employed in the method developed by Elsmore (1967) which was used for all the measurements described in Chapter VII.

The circuit is shown in Figure 16. The termination in this case is a semiconductor diode which is switched off and on at a frequency of 275 Hz by a signal from a square wave oscillator. The transmitted signal is of the form $V_0 \cos(\omega t-kx)$ and the reflected signal will then be equal to $AV_0 \cos(\omega t-k(2l_0-x)+\phi)$ where $A$ is the attenuation and $\phi$ is either 0 or $\pi$ depending on the state of the diode.
The input to the mixer is then of the form:

\[ V = V_0 (\cos(\omega t - kx) \pm A \cos(\omega t - k(2l_0 - x))) \]  \hfill (1)

This is fed into a phase-switched receiver (Ryle 1952) which is also driven by the 275 Hz oscillator. The phase switched receiver has the property that its output voltage is proportional to the scalar product of the steady and modulated voltage, so that:

\[ \text{Output} \propto V_0^2 A \cos(\omega t - kx) \cos(\omega t - k(2l_0 - x)) \propto AV_0^2 \cos 2k(l_0 - x) \]

The phase switched receiver is here being used to compare the phases of the modulated and unmodulated signal and so the output has nodes at distances of \( \frac{n\lambda}{4} \) from the termination. For the experiments described in Chapter VII a 70 MHz commercial crystal oscillator was used with a
frequency stability of better than 10 Hz per month, and
the only serious cause of error was the effect of secondary
reflections of the modulated signal from the oscillator
side of the slotted line. These were reduced by inserting
a 10 dB attenuator between these components (Figure 16)
but throughout the observations it was necessary to ensure
that the connections in this part of the circuit were left
undisturbed between readings. The usual method of taking
readings was to measure the output voltage at several positions
of the carriage then monitor the output on a chart recorder.
Changes of length of 1 mm were several times larger than the
noise level on the chart even for a half mile cable which,
at 70 MHz, produces a net attenuation of about 20 dB for the
reflected wave relative to the transmitted one.

The apparatus described in this section is suitable
only for monitoring the changes of electrical length of a
moderately long cable, and not for determining it absolutely.
The next section describes the development of a more sophis-
ticated system capable of measuring the total length of a
much longer cable.


The Cambridge Five Kilometre Telescope is now under
construction. This project requires the knowledge of the
electrical lengths of a very large number of R.F. cables both for sending local oscillator signals to the eight paraboloids from the main laboratory and for returning the signal at I.F. The variations in the lengths of the L.O. cables have been discussed in Chapter VII. The variations in the I.F. cables are less troublesome but their absolute lengths, and those of the path compensating cables, must be known to a small fraction of a wavelength at 45 MHz. The lengths of these cables range up to 2.5 km which means that the reflected signal is greatly attenuated: at 45 MHz it is over 50 dB down on the transmitted wave. The detection of such a weak signal and the measurement of its phase to the required accuracy is the object of the machine described in this section.

One alternative is to measure the electrical length at a lower frequency with correspondingly less attenuation. It is possible to show that for any particular type of cable there is an optimum frequency \( \nu \) for making such a measurement. For a cable of length \( L \) the amplitude of the reflected wave is proportional to \( \exp(-2\alpha \sqrt{\nu L}) \) where \( \alpha \) is a constant for the type of cable. The sensitivity with which the position of a node can be determined will therefore be proportional to \( \nu \exp(-2\alpha \sqrt{\nu L}) \). This function can be differentiated with respect to \( \nu \) to find the frequency
at which the sensitivity is greatest. It turns out that
the optimum frequency is inversely proportional to the
square of the cable length. For 100m of HM4 it is 3 GHz,
for 3 km it is 3 MHz.

In the event it was decided to build the machine to
work at 45 MHz, which gives reasonable sensitivity over
a wide range of cable length without making the distance
between nodes too long. It also a great advantage to make
measurements at the frequency at which the cables are to be
used, as the discontinuities in the cable will then have
exactly the same effect during tests as in operation. The
recent misfortunes at the site of the Westerbork radio
telescope in the Netherlands, an array similar to the One-
Mile Telescope, show the wisdom of this decision (Hamaker
1969). The cables in that system were measured at a
frequency very different to that at which they were to be
used and were buried before it was discovered that many
had strong resonances close to the 30 MHz I.F. frequency
in use at Westerbork, and had to be re-excavated. Although
it is very much hoped that no such drastic behaviour will
affect the cables in the 5 km telescope it is clear that
the electrical length of a cable can depend on frequency
and that there is a great advantage in working at 45 MHz
if possible.
As stated above, the reflected signal is about 50 dB fainter than the transmitted one at this frequency. The principle behind the new machine is to employ a directional coupler so that the signals reflected from the diode are preferentially fed into the receiver. This idea was suggested by Professor Ryle, but the development of the machine is the work of the author.

It can be shown that a 3 dB coupler is the most efficient for this job, and the quarter-wave hybrid (Figure 17) is an ideal example of such a device. This has been described by Smith (1961) although some of his equations do not appear to be quite accurate.
The oscillator and receiver are effectively isolated from each other. The power from the oscillator is split equally between the cable and the dummy load \( Z_D \), whereas the signal reflected from the diode is split equally between the oscillator and the receiver. An improvement of up to 30 dB in the ratio of reflected to transmitted signal can easily be obtained by this method, using a 'line-stretcher' to find the nodes, rather than a slotted line. The hybrid has the difficulty, however, that the phase of the leaking, unmodulated signal from the oscillator to the receiver is not well defined and depends on the small difference between the impedances of the cable \( Z_C \) and of the dummy load \( Z_D \). This is crucial since, as explained above, the phase-switched receiver operates by comparing the phases of a modulated and unmodulated signal. The operation of the hybrid circuit can most easily be discussed by noting that the problem of measuring the length of the cable can be reduced to that of measuring the phase of \( \Delta Z_C \) where \( \Delta Z_C \) is the change in the cable impedance when the diode is turned off and on. The precise connection between \( \Delta Z_C \) and the length of the cable is discussed later in this section.

It is possible, though tedious, to show that for a hybrid near its operating frequency and nearly matched;
\[ i_R = I_0 \left[ \frac{Z_C}{Z_0} \left( \frac{1}{4} - jT \right) \right] - \frac{Z_D}{Z_0} \left( \frac{1}{4} + jT \right) + jT \] (3)

\[ Z_0 \text{ is the nominal impedance of the hybrid and } T \text{ represents the fraction of the total current } I_0 \text{ that would be transmitted across the terminals of the hybrid if } Z_C \text{ and } Z_D \text{ were exactly equal. } T \text{ is typically of the order of } 0.03 \text{ but is a function of frequency of the form;} \]

\[ T(\nu) = T_0 - \frac{\pi}{4\sqrt{2}} \frac{(\nu - \nu_0)}{\nu_0} \] (4)

This equation need only be considered when the operating frequency is changed. For many purposes equation 3 may be approximated to;

\[ i_R = I_0 \left[ \frac{Z_C - Z_D}{4Z_0} + jT \right] \] (5)

It can be shown that when the impedance of the cable is modulated by the diode the output \( (V_C) \) from the phase-switched receiver is given by;

\[ V_C = kI_0^2 \left( \frac{\Delta Z_C}{Z_0} \right) \left( \frac{1}{4} - jT \right) \left[ \frac{Z_C}{Z_0} \left( \frac{1}{4} - jT \right) - \frac{Z_D}{Z_0} \left( \frac{1}{4} + jT \right) + jT \right] \] (6)

where \( \bar{Z}_C \) is the mean value of \( Z_C \) during the two parts of the cycle. It is evident that the term in square brackets in equation 6 is not purely a function of the machine since it depends on \( \bar{Z}_C \), a property of the cable under test which can vary significantly from one sample to
another. The problem was solved by inserting a circuit to produce a modulated calibrating signal with a phase which could be compared with that of the signal reflected from the diode. This is done by keeping \( Z_C \) constant and equal to \( Z_D \) and varying \( Z_D \) by a small amount \( \Delta Z_D \) about the value \( Z_D \). The output from the receiver in this case is given by;

\[
V_D = kI_0^2 \left( \frac{\Delta Z_D}{Z_o} \right) \left( \frac{1}{4} + jT \right) \left[ \frac{Z_C}{Z_o} \left( \frac{1}{4} - jT \right) - \frac{Z_D}{Z_o} \left( \frac{1}{4} + jT \right) + jT \right]
\]  

(7)

The terms in the square brackets in equations 6 and 7 are identical so long as \( Z_D \) in equation 6 is chosen to be equal to \( Z_D \) in equation 7 and \( Z_C \) in equation 7 is chosen to be equal to \( Z_C \) in equation 6. This is done in the latter case by replacing the diode by a matched load when \( Z_D \) is being modulated. The same thing can be done more easily by using a PIN diode as the termination, whose impedance can be changed from 0 to 75 ohms to \( \infty \) simply by passing a known current through it. The phase of \( \Delta Z_C \) can thus be compared with the phase of \( \Delta Z_D \), which is known and is built into the apparatus. Then, as far as phase is concerned;

\[
\frac{\Delta Z_C}{Z_o} \left( \frac{1}{4} - jT \right) = \frac{\Delta Z_D}{Z_o} \left( \frac{1}{4} + jT \right)
\]  

(8)

\( jT \) is a constant for the apparatus, but in any case represents only a small phase term. The operation of the
instrument involves first modulating $Z_D$ and measuring the phase between $\Delta Z_D$ and the rest of the right-hand side of equation 7, and then modulating $Z_C$ and adjusting the length of the cable until the same phase angle results. The measurement of the phase angle may be carried out using the circuit in Fig 18.

The receiver is a Phase-Switch F receiver (Elsmore et al. 1966). This is effectively two phase-switched receivers in parallel, with their outputs subtracted. The two inputs to the receivers are fed from a second
hybrid $H_2$, while the signal from the oscillator is split by a third hybrid $H_3$ so that a constant phase unmodulated signal $i_A$ is available at $H_2$ of approximately the same size as $i_R$. $\Delta i_R$ is the small variation in $i_R$ due to the modulation of either $Z_D$ or $Z_C$. Now the inputs to the receivers are given by:

$$i_1 = i_A + (i_R \pm \Delta i_R)$$
$$i_2 = i_A - (i_R \pm \Delta i_R)$$

(9)

The output $V$ is given by:

$$V = V_1 - V_2 = k \Delta i_R (G_1(i_R + i_A) - G_2(i_R - i_A))$$

(10)

where $G_1$ and $G_2$ are the gains of the two I.F. strips of the receiver. Unfortunately it is not possible to set $G_1 = G_2$ in this application, since $i_A$ and $i_R$ are very highly correlated signals and the power in each I.F. strip is very different. This means that the A.G.C. circuits on the two sides of the receiver will adjust the gains to different values even if they appear to be equal. However it can be shown that by a suitable choice of the phase angle between $\Delta Z_D$ and $i_A$, which is determined simply by the lengths of various bits of cable inside the instrument, it is always possible to set $G_1$ and $G_2$ such that $V = 0$ and $\Delta i_R$ is in quadrature with $(G_1(i_A + i_R) - G_2(i_A - i_R))$. The necessary condition is that $PQ = \frac{\lambda}{4}$ and that $|i_A| > |i_R|$. The ratio of the gains having thus been set by making $V = 0$, $Z_D$ is
made constant at $\bar{Z}_D$ and the diode at the end of the cable is modulated while the length of the cable is adjusted for a null again. This process is very quick and involves no delicate tuning or fine adjustment and, since the unmodulated currents $i_A$ and $i_R$ are identical whether $\Delta Z_C$ or $\Delta Z_D$ are being measured, does not depend on knowledge of the linearity of the receivers. This system has been built and tested with 1.5 miles of cable at 45 MHz. The precision of the null-point determination was limited only by the line-stretcher and was considerably less than 1 mm. As the equipment stands it has the slight disadvantage that the line stretcher needs to be at the far end of the cable from the hybrid for the most rapid work. This is because internal reflections in the line stretcher will affect $\bar{Z}_C$ in equations 6 and 7 if the stretcher is close to the hybrid. This does not affect the final accuracy since it is with $\Delta Z_D$ that $\Delta Z_C$ is compared, but means that slightly more complicated adjustments of $G_1$ and $G_2$ may be required. Ways of getting round this problem are being investigated.

A second method of adjustment of the instrument has been suggested by Ryle (1969) which requires the use of only one of the two I.F. strips, and does not involve modulation of $Z_D$. Ryle proposes switching off $i_A$ and tuning $Z_D$ for a minimum in the power transmitted across the hybrid, then
using the phase-switched receiver to compare $\Delta i_R$ with $i_A$, thus eliminating the steady current $i_R$ altogether. Ryle's method relies for its accuracy on a rejection of the signal leaking through the hybrid of 50 dB. An experiment is planned to see how easy this is to maintain and to see which method is more convenient. In practice, especially if the line stretcher is to be near the hybrid, Ryle's method is likely to be much slower since the simultaneous adjustment of two parameters is required each time the stretcher is moved, whereas in the system proposed here only the relative gains of the sets need be altered - an operation requiring the adjustment of only one knob.

It has been stated that the precision of measurement using this instrument is better than 1 mm. Experiments with measuring the same length of cable from either end, however, disclosed an inconsistency much greater than this, typically of the order of 5 mm, and the cause of this required a rather more careful analysis of the relation between $\Delta Z_C$ and the electrical length of a cable.

\[ \begin{array}{ccc}
A & \rightarrow & C \\
\leftarrow B & - & - & - & - & \rightarrow D \\
Z_1 & - & - & - & - & Z_2
\end{array} \]

FIG. 19.
From consideration of the boundary conditions at every discontinuity it can be shown with complete generality that at a single frequency it is possible to express the relationship between the amplitudes $A$, $B$, $C$ and $D$ of the travelling waves (Figure 19) at the ends of a lossy, discontinuous cable of varying impedance by the equation;

$$\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} K_0 & M_0 \\ N_0 & L_0 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix}$$  \hspace{1cm} (11)

where $K_0 L_0 - M_0 N_0 = Z_1 / Z_2$ \hspace{1cm} (12)

$Z_1$ and $Z_2$ are the travelling wave impedances at the two ends of the cable, not necessary equal, and $L_0$ is what we are trying to measure; its phase gives the length of the line and its amplitude the attenuation ($|L_0| > 1$). It can be seen that if $D = 0$ then $B = -(N_0 / L_0)A$, and that if $A = 0$ then $C = (M_0 / L_0)D$. $M_0$ and $N_0$ are therefore complex reflection coefficients arising from the discontinuities at various points in the line. $M_0$ and $N_0$ are in general much smaller than $L_0$, but, as will be shown, they are the cause of the limiting accuracy in any reflection method of cable measuring.

Let the line be terminated by a diode with on and off impedances $Z_N$ and $Z_F$ and define;

$$\zeta_1 = \frac{Z_N - Z_2}{Z_N + Z_2} \quad \text{and} \quad \zeta_2 = \frac{Z_F - Z_2}{Z_F + Z_2}$$  \hspace{1cm} (13)

where $Z_2$ is, as before, the impedance at one end of the line.
It is then possible to show that when $\xi$ changes at one end of the cable, due to the modulation of the diode, the change of impedance $\Delta Z_0$ is given by:

$$
\Delta Z_0 = \frac{2Z_2}{L_0} (\xi_1 - \xi_2) \left[ 1 - \frac{2N_o}{L_0} + (\xi_1 + \xi_2) \frac{M_o}{L_0} \right]
$$

(14)

For a perfect diode $\xi_1 = -1$ and $\xi_2 = +1$, so that

$$
\Delta Z_{C1} = \frac{4Z_2}{L_0^2} \left[ 1 - \frac{2N_o}{L_0} \right]
$$

(15)

If the cable is reversed and the measurement repeated;

$$
\Delta Z_{C2} = \left( \frac{Z_2}{Z_1} \right) \frac{Z_2}{L_0^2} \left[ 1 + \frac{2M_o}{L_0} \right]
$$

(16)

Thus the spurious reflections inside the cable, represented by the terms $M_o$ and $N_o$ are the cause of the errors in the measurement of $L_0$, since the phase of $\Delta Z_0$ in the above two equations depends on them. An error of $2^\circ$ phase at $45^\circ$ might arise due to a VSWR of only 1.003 in the cable which is by no means unreasonably large.

It is possible to eliminate this error entirely, however, by measuring the cable from both ends and then repeating the readings with the diode switching between hard on and matched (75 ohms) rather than hard on and hard off. In this case $\xi_1 = 0$ and $\xi_2 = -1$. Applying equation 14 we get;

$$
\Delta Z_{C3} = \frac{2Z_2}{L_0^2} \left[ 1 - \frac{2N_o}{L_0} - \frac{M_o}{L_0} \right]
$$

(17)
and on reversing the cable again;

$$\Delta Z_{G4} = 2 \left( \frac{Z_2}{Z_1} \right) \frac{Z_2}{L_o} \left[ 1 + \frac{2M_o}{L_o} + \frac{N_o}{L_o} \right]$$  \hspace{1cm} (18)

From these last four equations $M_o$ and $N_o$ can be eliminated and $L_o$, and hence the length of the cable, can be determined. This has not yet been checked experimentally, as a new termination has to be constructed which uses a PIN diode, whose R.F. impedance is a known function of the DC current through it. It is anticipated that with this modification the machine described in this chapter will be able to measure the lengths of cables more than 2.5 km long with an accuracy of about 1 mm. This is considerably better than is required for installing the I.F. and path compensating cables in the Five Kilometre Telescope, and it may not prove necessary to measure the cables more than once. For the highest accuracy, however, allowance must be made for the imperfections of the reflecting diode and equation 13 should be used to derive the correct value of $Z_2$ when the reversed biased diode has self-capacity.

With the method described in this chapter there is still an ambiguity of the number of half-wavelengths of cable up to the node being measured. This can be resolved by tuning the oscillator to the frequency at which the next null occurs with the same length of cable. The frequency
response of the hybrid (equation 4) is now a significant factor and it can be shown that this has the effect of making the cable seem shorter than it really is, by a fixed fraction of a wavelength. If the frequency is changed by $\delta \nu$ to obtain the next null, then the length of the cable is:

$$l_0 = \left( \frac{\nu_0}{\delta \nu} + \sqrt{2} \right) \frac{\lambda}{4}$$

(19)

$\delta \nu$ can be measured to the required accuracy by a frequency meter, but it is likely that in many cases the number of wavelengths may be most easily estimated with a tape measure.

The final design of the completed instrument is awaiting decisions as to its mode of use in conjunction with contractors for the Five Kilometre Telescope, but the project has demonstrated that accurate measurements of the very long lengths of cable necessary to operate large radio telescopes is a feasible and relatively simple procedure.
REFERENCES


Curtis, H.D., 1918, Pub. Lick Obs., 13 57


Gottlieb, D.M. and Upson II, W.L., 1968, 
Maryland Astron. J., 1, No. 2, 10.
Holden, D.J. and Caswell, J.L., 1969,
Kellerman, I.K. and Pauliny-Toth, I.I.K., 1967,
quoted by Terzian (1968).
Kundu, M.R. and Velusamy, T., 1968,
Liller, W. and Shao, C-Y., Planetary Nebulae (I.A.U.
Symposium No. 34) ed. D.E. Osterbrock and C.R. O'Dell
Locke, J.L., Galt, J.A. and Costain, C.H., 1964,
Mathews, W.G., 1968, Interstellar Ionised Hydrogen,
Mezger, P.G., 1968, Interstellar Ionised Hydrogen,
Mezger, P.G., Altenhoff, W., Schraml, J., Burke, B.F.,
Reifenstein III, E.C. and Wilson T.L., 1967b,
Mezger, P.G., Schraml, J. and Terzian, Y., 1967a,
Neville, A.C., Windram, M.D. and Kenderdine, S. (1969),
In the press.
Osterbrock, D.E. et al., 1968, Reported Discussion.
Planetary Nebulae (I.A.U Symposium No 34),
ed. D.E. Osterbrock and C.R.O'Dell (D. Reidel,
Palmer, P., Zuckerman, B., Penfield, H. and Lilley, A.E.
Pauliny-Toth, I.I.K. and Shakeshaft, J.R. 1962,
Pauliny-Toth, I.I.K., Wade, C.M. and Heeschen, D.S., 1966,
Pilkington, J.D.H. and Scott, P.F., 1965,
(Benjamin, New York) p. 87.
Rogers, A.E.E., Moran, J.M., Crowther, P.P., Burke, B.F.,
Meeks, M.L., Ball, J.A. and Hyde, G.M., 1967,
Ryle, M., Elsmore, B. and Neville, A.C., 1965,
Sato, F., Akabane, K. and Kerr, F.J., 1967,
Snyder, L.E., Buhl, D., Zuckerman, B. and Palmer, P., 1969,
Spitzer, L., 1968a, Nebulae and Interstellar Matter,
ed. B.M. Middlehurst and L.H. Aller (Chicago
———, 1968b, Diffuse Matter In Space, (Interscience,
New York) p. 188.
and Propogation) 9 75.
———, 1968, Planetary Nebulae (I.A.U. Symposium No. 34)
ed. D.E. Osterbrock and C.R. O'Dell (D. Reidel,
Dordrecht-Holland) p. 87.

and, finally

Langbridge, F., 1896, A Cluster of Quiet Thoughts (Religious Tract Society).
Appendix A. Modelfitting Programs.

The use of modelfitting programs has been alluded to several times in this dissertation. These have been used for two purposes, firstly to study the apparent distortions in shape of a source produced by the One Mile Telescope at low declinations and secondly to compare source maps of HII regions with those of models comprising gas distributions of varying optical depth. This appendix describes these programs in slightly more detail than is done in the main body of the dissertation.

In the first of these programs the model source may be specified either as a 25 x 25 matrix of points or as an analytic function of position in either two or three coordinates. In the latter case the program integrates along the line of sight, so as to turn a three dimensional model into a two dimensional brightness distribution. If the source is an HII region the model may be defined simply in terms of the electron temperature and density expressed as functions of position; the appropriate allowances for self absorption are then made automatically.

The brightness temperature distribution thus defined is convolved with a function representing the known beamshape

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of the telescope at the appropriate frequency and declination, and the resultant map is then plotted out by the computer in aerial or sky coordinates on any chosen scale. The map before convolution may also be plotted and a line-printed version of both maps is produced in addition. In the case of HII regions the contours represent known increments in brightness temperature, and, if the distance to the HII region is specified, its total flux density at the observing frequency will be calculated. In this form the program was used in the study of DR 21 and NGC 7027 together with a simplified version which merely calculated the integrated emission from the same models at a number of specified frequencies. This saved computing time as a large number of models could be eliminated simply because their spectra did not agree with that of the real source.

The modelfitting program described here is very flexible and has been used by other members of the department for work not connected with HII regions, but it is rather slow, requiring about 30 secs. computing time for each model. In many cases where an estimate of the size of a source is needed, however, computing a complete map for each model produces a great deal of redundant information, especially when the model has a lot of symmetry. A second program was therefore written which was concerned solely with the
half-widths of spherically symmetric sources, in particular those comprising a sphere of gas of uniform temperature and density. For any chosen source diameter, expressed as a fraction of the half-power beamwidth, the half-width of the map was calculated for different values of the optical depth through the centre of the source. For each declination a series of graphs was then plotted showing the relationship between the actual source diameter and the apparent diameter, both normalised to the half-power beamwidth, for optically thin, optically thick and intermediate cases.

This program was used for W 49A and for two of the larger components of NGC 6857, and is two orders of magnitude faster than the mapping program described above. By confining attention to the main part of the source only, and not to the low brightness perimeters it was possible to accelerate the mathematics by approximating the known beam of the telescope to a Gaussian profile, which it very much resembles except at the edges.

The results obtained with this program showed that Gaussian widths are a very unreliable guide to the sizes of HII regions, since the ratio of Gaussian width to source diameter depends very much on both the sizes of the source and the optical depth through its centre. The use of a
constant conversion factor (Mezger and Henderson 1967) is only accurate when the source diameter is much larger than the beamwidth and when there is no self absorption.

This dissertation has, naturally, taken many weeks to write, and in several cases modifications have had to be made during its writing as new data were published by other astronomers. In most cases, however, I have not been able to incorporate results which have appeared since the beginning of July 1969, and in this appendix the most recent data, all unpublished, which are relevant to the contents of the dissertation are briefly described. In three cases they concern NGC 6857 and it is interesting how much attention has been suddenly paid to this object in recent months.

(1) Professor B.F. Ney of the University of Minnesota has reported (private communication) that there is weak but not remarkable infrared emission from the vicinity of NGC 6857. It is not yet known how it compares with that from M33, which is closer to the sun, or from nearby more distant galaxies, if any, it emanates.

(2) Professor Ney also showed us a proposed paper by A. H. Rubin and B. E. Kirshner of 1969 which he has read.
Appendix B. Very Recent Data

This dissertation has, naturally, taken many weeks to write, and in several cases modifications have had to be made during its writing as new data were published by other astronomers. In most cases, however, I have not been able to incorporate results which have appeared since the beginning of July 1969, and in this appendix the most recent data, all unpublished, which are relevant to the contents of the dissertation are briefly described. In three cases they concern NGC 6857 and it is interesting how much attention has been suddenly paid to this object in recent months.

(1) Professor E.P. Ney of the University of Minnesota has reported (private communication) that there is weak, but not remarkable infra-red emission from the vicinity of NGC 6857. It is not yet known how it compares with that from DR 21, which is closer to the sun, or from which condensation, if any, it emanates.

(2) Professor Ney also showed me a preprint of a paper by R. H. Rubin and B. E. Turner of NRAO. This describes
their independent 15.4 GHz observations of the NGC 6857 group of sources with the 140' dish with a beamwidth of 2'arc, ten times that of my observations in Chapter IV. Their total flux density at 15 GHz is close to that of Davies indicating that the whole source is optically thin above 7 GHz. as predicted in Chapter IV. Although the four components are not distinguishable in the NRAO map it is evident that, compared with Figure 10, a disproportionate amount of the emission is centred on Component A (K 3-50), confirming that at 2.7 GHz it is still optically thick, as assumed in section IV 5.

Rubin and Turner have observed the H109α and H137β lines in the source and remark on their unusual width, indicative, as they say, of several components or of rapid expansion velocities. Formaldehyde and HI absorption and OH emission are also described in this paper and the line frequencies all tend to confirm the kinematic distance of 8 - 9 kpc. This American data is thus complimentary to the high resolution work described in Chapter IV and it is most satisfying that Rubin and Turner have independently come to the conclusion that K 3-50, at least, is more likely to be an HII condensation than a planetary.

(3) I have been sent a preprint of a paper on the
21 cm absorption profiles from DR 21 and the NGC 6857 sources by A.R. Thompson, R.S. Colvin and M.P. Hughes of Owens Valley. The NGC 6857 data confirm the distance to the source, but it is the DR 21 data which is the more interesting since, although a distance uncertainty still exists, the authors show that it might be possible to explain some of the unusual features in the HI spectrum by assuming the presence of a expanding shell of neutral hydrogen which is being compressed and forced outwards by the shock wave accompanying the ionisation front. This is the first time direct evidence for the existence of a high velocity HI shell around an HII region, compact or diffuse, has been found.

(4) In answer to a query about a point in his paper (section 2 above) I have received a letter from Dr. Rubin describing recent NRAO observations of NGC 7027 by Terzian and Balick and others. In these the 5GHz H 109α was not found, after much longer integration times than those of Mezger et al. (1967b). The discussion of the discrepancy in electron temperature measurements in Chapter V, section 4 is therefore no longer relevant, but clearly either the theory of recombination lines or the generally accepted ideas about NGC 7027 will need drastic revision in light of this result.
Plate I. Reproductions of the blue (above) and red (below) prints of the Sky Survey around NGC 6857, which is the nebula visible in the blue print. The positions of components A, B and C are marked and the large nebula in the red print is Sharpless 100. North is at the top, east is on the left.
Plate II. NGC 7027 photographed in Hα light with the 200" telescope by Minkowski. Reproduced from "Nebulae and Interstellar Matter", ed. Middlehurst and Aller (1968). North is at the top and east is to the left. The black line underneath the photographs is approximately 20" arc long.