Chapter 12

Dark Matter in Elliptical Galaxies?

By analogy with disk galaxies, one might expect that the velocity dispersion profile of an elliptical galaxy can be analyzed to inferr the total mass and the amount of dark matter as functions of radius. In fact, dispersion alone is *not* enough; without additional constraints on the shape of stellar orbits, a very wide range of mass profiles are consistent with a given dispersion profile. Luckily, there is one component commonly found in elliptical galaxies which can be used to detect extended dark halos – X-ray emitting gas.

12.1 The Jeans Equations

To reduce the collisionless Boltzmann equation to something more manageable, we can project out the velocity dimensions. There are many ways to do this (BT87, Chapter 4.2); the first of these is simply to integrate over all velocities:

$$\int d\mathbf{v}\,\frac{\partial f}{\partial t} + \int d\mathbf{v}\,v_i\frac{\partial f}{\partial r_i} - \int d\mathbf{v}\,\frac{\partial \Phi}{\partial r_i}\frac{\partial f}{\partial v_i} = 0\,. \tag{12.1}$$

Rearranging the first two terms, and using the divergence theorem (BT87, Eq. 1B-44) to get rid of the third term, the result is a continuity equation for the 3-D stellar density,

$$\frac{\partial \nu}{\partial t} + \frac{\partial}{\partial r_i}(\nu\overline{v_i}) = 0\,, \tag{12.2}$$

where the stellar mass density and momentum density are

$$\nu \equiv \int d\mathbf{v}\,f\,, \qquad \nu\overline{v_i} \equiv \int d\mathbf{v}\,f v_i\,, \tag{12.3}$$

with the integrals taken over all velocities.

To obtain the next (three) equations in this series, multiply the CBE by the velocity component v_j, and integrate over all velocities. Proceeding much as above, the result is

$$\frac{\partial}{\partial t}(\nu\overline{v_j}) + \frac{\partial}{\partial r_i}(\nu\overline{v_j v_i}) + \nu\frac{\partial \Phi}{\partial r_j} = 0\,. \tag{12.4}$$

where

$$\nu\overline{v_j v_i} \equiv \int d\mathbf{v}\,f v_j v_i\,. \tag{12.5}$$

This may be placed in a 'more familiar' form by defining the dispersion tensor

$$\sigma_{ij}^2 \equiv \overline{v_j v_i} - \overline{v_j}\,\overline{v_i} \tag{12.6}$$

which represents the distribution of stellar velocities with respect to the mean at each point. The result is called the equation of stellar hydrodynamics,

$$\nu \frac{\partial \overline{v_j}}{\partial t} + \nu \overline{v_i} \frac{\partial \overline{v_j}}{\partial r_i} = -\nu \frac{\partial \Phi}{\partial r_j} - \frac{\partial}{\partial r_i}(\nu \sigma_{ij}^2)\,, \tag{12.7}$$

because it resembles Euler's equation of fluid flow, with the last term on the right representing an anisotropic pressure. Note that because there is no equation of state, this pressure is not related in any simple way to the mass and momentum density defined in (12.3).

By multiplying the CBE by $v_i v_j \ldots v_l$ and integrating over all velocities, yet higher order equations can be formed; each, alas, makes reference to quantities of one order higher than those it describes. To close this hierarchy of equations, one must make some physical assumptions.

Caveat: Any *physical* solution of the CBE must have a non-negative phase-space distribution function. The velocity moments of the CBE describe possible solutions, but do *not* guarantee that a non-negative $f(r,v;t)$ exists. You've been warned!

12.2 Velocity Dispersions in Spherical Systems

Perhaps the simplest application of the equations of stellar hydrodynamics is to equilibrium spherical systems. Assuming that all properties are constant in time and invariant with respect to rotation about the center, the radial Jeans equation is

$$\frac{d}{dr}(\nu \overline{v_r^2}) + \frac{2\nu}{r}(\overline{v_r^2} - \overline{v_t^2}) = -\nu \frac{d\Phi}{dr}\,, \tag{12.8}$$

where v_r and v_t are speeds in the radial and tangential directions, respectively. Define the radial and tangential dispersions,

$$\sigma_r^2 \equiv \overline{v_r^2}\,, \qquad \sigma_t^2 \equiv \overline{v_t^2}\,, \tag{12.9}$$

and the anisotropy profile,

$$\beta(r) \equiv 1 - \sigma_t^2/\sigma_r^2\,, \tag{12.10}$$

which is $-\infty$ for a purely tangential velocity distribution, 0 for an isotropic distribution, and $+1$ for a purely radial velocity distribution. Then (12.8) becomes

$$\frac{1}{\nu}\frac{d}{dr}(\nu\sigma_r^2) + \frac{2}{r}\beta\sigma_r^2 = -\frac{d\Phi}{dr}\,. \tag{12.11}$$

If $\nu(r)$, $\beta(r)$, and $\Phi(r)$ are known functions, (12.11) may treated as a differential equation for the radial dispersion profile. Thus one application of this equation is making models of spherical systems. From an observational point of view, however, $\nu(r)$ and $\sigma_r(r)$ may be known, and the goal is to obtain the mass profile, $M(r)$. Using the relation

$$\frac{d\Phi}{dr} = G\frac{M(r)}{r^2}\,, \tag{12.12}$$

the mass profile is given by

$$M(r) = -\frac{\sigma_r^2 r}{G}\left(\frac{d\ln\nu}{d\ln r} + \frac{d\ln\sigma_r^2}{d\ln r} + 2\beta(r)\right)\,. \tag{12.13}$$

The catch here is that we don't know $\beta(r)$. One option is to assume that the velocity distribution is isotropic, so that $\beta = 0$ for all r. This allows us to calculate mass profiles, but the results are uncertain because there is no good reason why velocity distributions should be isotropic. Another option is to assume that the galaxy has a constant mass-to-light ratio, and calculate the anisotropy profile $\beta(r)$ needed to satisfy (12.13) (Binney & Mamon 1982).

12.2.1 Case study: M87

Early studies of M87 (= NGC 4486) *assumed* an isotropic velocity distribution, and used measurements of surface brightness and line-of-sight velocity dispersion to try and measure the mass profile; the results suggested the presence of a large unseen mass – presumably a black hole – at small radii (Sargent et al. 1978). But Binney & Mamon (1982) showed that the observations could be fit by a radially anisotropic velocity distribution with no unseen massof any kind. van der Marel (1994) reaffirmed this point by fitting the data with two kinds of models: anisotropic models with no black hole and isotropic models with a black hole. Now that observations of a disk of ionized gas at the center of M87 have established the presence of a black hole of mass $M_\bullet \simeq 2.4 \times 10^9\,\mathrm{M}_\odot$ (Harms et al. 1994) one may turn the analysis around and try to estimate the degree of velocity anisotropy required. Dressler & Richstone (1990) and Merritt & Oh (1997) have shown that the relatively flat dispersion profile of M87 can be reconciled with a supermassive black hole if the velocity distribution is strongly biased in the tangential direction near the center of M87.

12.2.2 Case study: NGC 4697

Recently the kinematics of the E6 galaxy NGC 4697 have been probed out to nearly $3R_\mathrm{e}$ by line-of-sight velocity measurements of 535 planetary nebulae (Méndez et al. 2001). The data show σ falling in a nearly Keplerian fashion, inviting the speculation that this galaxy has no dark halo. As an illustration, a spherical Hernquist (1993) model provides an excellent fit to the data, yielding a blue mass-to-light ratio $M/L \simeq 11$ independent of radius. But this fit *assumes* that the PN progenitors have isotropic velocities; this assumption can be ruled out, as the next section will show.

12.3 Axisymmetric Systems

Since many elliptical galaxies show some degree of flattening, it seems reasonable to try – at a minimum – an axisymmetric model based on stellar hydrodynamics. Working in cylindrical coordinates (R,ϕ,z), the time-independent axisymmetric version of (12.7) is

$$\frac{\partial}{\partial R}(\nu\overline{v_R^2}) + \frac{\nu}{R}(\overline{v_R^2} - \overline{v_\phi^2}) + \frac{\partial}{\partial z}(\nu\overline{v_\mathrm{R}v_\mathrm{z}}) = -\nu\frac{\partial\Phi}{\partial R}\,, \qquad (12.14)$$

$$\frac{\partial}{\partial z}(\nu\overline{v_z^2}) + \frac{1}{R}\frac{\partial}{\partial R}(\nu\overline{v_\mathrm{R}v_\mathrm{z}}R) = -\nu\frac{\partial\Phi}{\partial z}\,. \qquad (12.15)$$

If we *assume* that the galaxy's distribution function has the form $f = f(E,J_z)$ then we can simplify the equations; the radial and vertical dispersions obey $\overline{v_R^2} = \overline{v_z^2}$ and $\overline{v_R v_z} = 0$, so

$$\frac{\partial}{\partial R}(\nu\overline{v_R^2}) + \frac{\nu}{R}(\overline{v_R^2} - \overline{v_\phi^2}) = -\nu\frac{\partial\Phi}{\partial R}\,, \qquad (12.16)$$

$$\frac{\partial}{\partial z}(\nu\overline{v_R^2}) = -\nu\frac{\partial\Phi}{\partial z}\,. \qquad (12.17)$$

At each R one can calculate the mean squared velocity in the R direction by integrating (12.17) downward from $z = \infty$; once $\overline{v_R^2}$ is known, (12.16) gives the mean squared velocity in the ϕ direction.

The Jeans equations don't specify how the azimuthal motion is divided into streaming and random components. One popular choice for the streaming velocity is

$$\overline{v_\phi} = \sqrt{k(\overline{v_\phi^2} - \overline{v_R^2})}\,, \tag{12.18}$$

where k is a free parameter (Satoh 1980). The dispersion in the ϕ direction is

$$\sigma_\phi^2 = \overline{v_\phi^2} - \overline{v_\phi}^2\,. \tag{12.19}$$

Note that if $k = 1$ the velocity dispersion is isotropic and the excess azimuthal motion is entirely due to rotation, while for $k < 1$ the azimuthal dispersion exceeds the radial dispersion.

12.3.1 Application to elliptical galaxies

Jeans equations models have been constructed of a number of elliptical galaxies (Binney, Davies, & Illingworth 1990; van der Marel, Binney, & Davies 1990; van der Marel 1991; van der Marel et al. 1994b; Cretton & van den Bosch 1999). The basic procedure is to:

1. Observe the surface brightness $\Sigma(x', y')$.
2. Deproject to get the stellar density $\nu(R, z)$, assuming an inclination angle.
3. Compute the potential $\Phi(R, z)$, assuming a constant mass-to-light ratio; terms representing unseem mass may be included.
4. Solve the Jeans equations for the mean squared velocities.
5. Divide the azimuthal motion into streaming and random parts using (12.18).
6. Project the velocities back on to the plane of the sky to get the line-of-sight velocity and dispersion $v_{\rm los}(x', y')$ and $\sigma_{\rm los}(x', y')$.
7. Compare the predicted and observed kinematics, adjust parameters to improve the match, and go back to step #2.

The inclination angle, mass-to-light ratio, unseen mass distribution, and rotation factor k are the unknown parameters to be determined by trial and error.

Some conclusions following from this exercise are that:

- Isotropic oblate rotators ($k = 1$) generally *don't fit* the data, even though some of the galaxies modeled lie close to the expected relation between v_0/σ_0 and ε.
- Some galaxies (*e.g.* NGC 1052, M32) *are* well-fit by two-integral models.
- In other cases, the predicted major-axis velocity dispersions are often larger than those observed.
- Much as in the case of disk galaxies, stellar kinematic measurements generally don't extend far enough in radius to offer strong evidence for dark halos.

If no parameter set yields a model matching the observations then presumably the underlying assumptions of the models are at fault. Within the context of two-integral models one may try to improve the fit by adding dark mass or by letting the rotation parameter k depend on radius. However, the overly-large major-axis dispersions noted above indicate a fundamental problem with two-integral models (Merrifield 1991). In such models, the vertical and radial dispersions are always equal. Thus the flattening of such a model is determined by the energy invested in azimuthal motion, so that $\overline{v_\phi^2} > \overline{v_R^2} = \overline{v_z^2}$; at points along the major axis, motion in the ϕ direction projects along the line of sight, increasing the predicted velocity dispersion. To account for the observed dispersions, we require $\overline{v_R^2} > \overline{v_z^2}$. This is only possible if the distribution function depends on a third integral.

Conicidentally, NGC 4697 is one of the galaxies which can't be modeled using this procedure; slits offset from the major axis show smaller rotation velocities than predicted by the models (Binney et al. 1990). This strongly suggests that NGC 4697 is *not* dynamically simple and is poorly represented by an isotropic model!

12.4 Hydrostatic Equilibrium of Hot Gas

There is one galactic component which is *known* to possess an isotropic velocity distribution: the hot gas responsible for extended X-ray emission in ellipticals. In this case the observable quantities are the gas density and temperature profiles, $n_\mathrm{e}(r)$ and $T(r)$, respectively. In terms of these the mass profile is

$$M(r) = -\frac{kT(r)r}{Gm_\mathrm{p}\mu}\left(\frac{d\ln n_\mathrm{e}}{d\ln r} + \frac{d\ln T}{d\ln r}\right) . \tag{12.20}$$

where k is Boltzmann's constant, m_p is the proton mass, and μ is the mean molecular weight (e.g. Sarazin 1987). The parallels between (12.13) and this equation are obvious.

This method has several advantages: (1) it requires no assumptions about velocity anisotropy, (2) it probes the mass distribution at radii where the stellar surface brightness is too low to measure velocity dispersions, and (3) with enough X-ray telescope time, good statistics can be accumulated, whereas tracers such as globular clusters and planetary nebulae are limited in number. But until recently very few galaxies were well enough resolved to yield detailed temperature profiles.

12.4.1 Case studies: NGC 4472

X-ray data for NGC 4472 (Trinchieri et al. 1986; Irwin & Sarazin 1996) can be used to constrain the mass distribution of this galaxy (Lowenstein 1992; Mathews & Brighenti 2003). The gas has $T \simeq 10^7\,\mathrm{K}$ and samples the potential out to nearly $10R_\mathrm{e}$. An analysis using (12.20) shows that a dark halo is clearly present beyond $\sim 1R_\mathrm{e}$ and may account for $\sim 90\%$ of the total mass within $10R_\mathrm{e}$. For an adopted blue mass-to-light ratio $M/L \simeq 7$, the stellar component dominates the mass at $0.1R_\mathrm{e}$; to account for the observed profile of velocity dispersion with radius, the stellar velocity distribution must be radially anisotropic, with $\beta \simeq 0.53$ out to nearly $1R_\mathrm{e}$.